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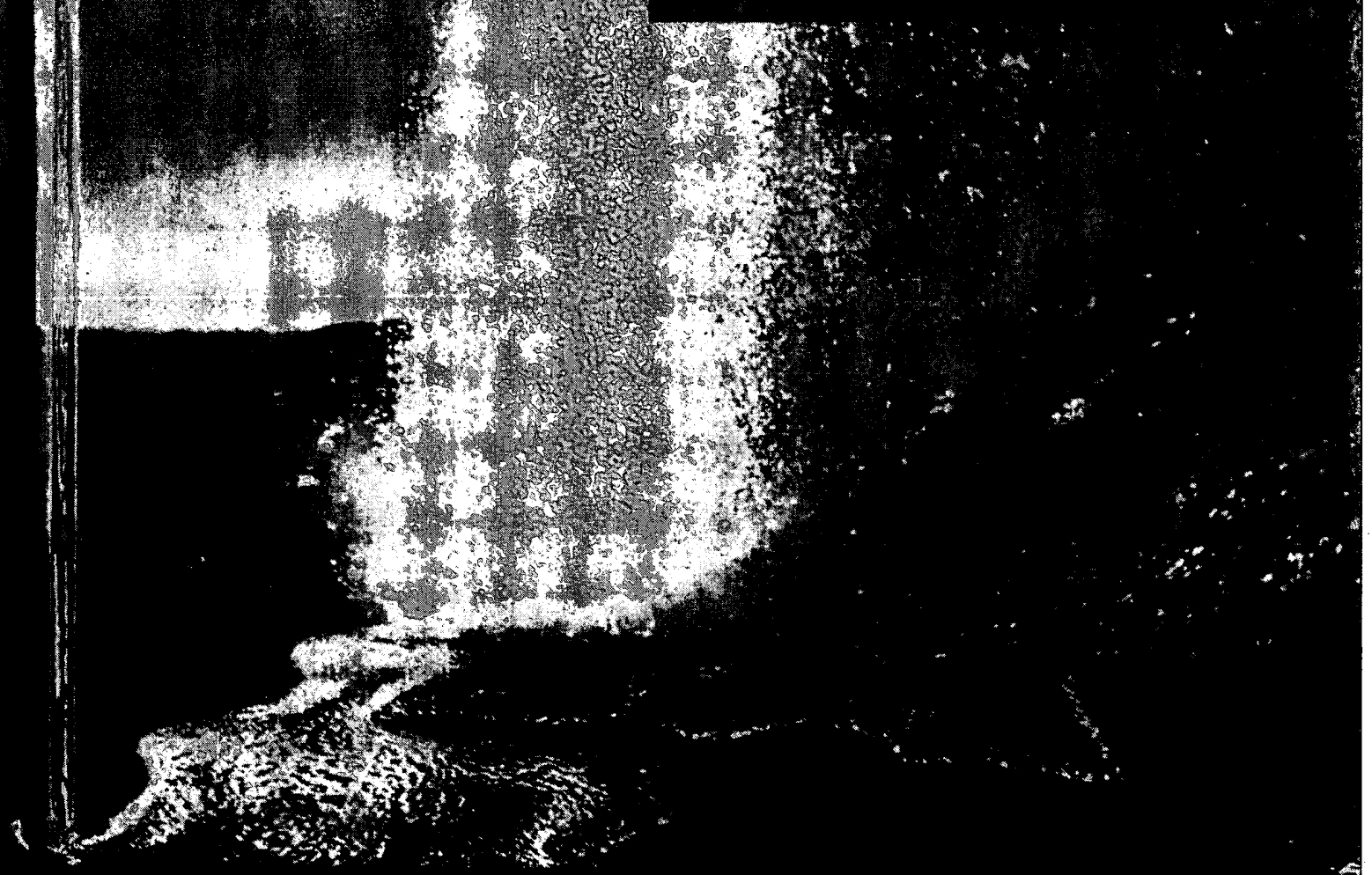
Heat Transfer

A Practical Approach

YUNUS A. ÇENGEL



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HEAT TRANSFER

A PRACTICAL APPROACH

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HEAT TRANSFER: A PRACTICAL APPROACH

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Heat Exchangers

CHAPTER 10

Heat exchangers are devices that facilitate the *exchange of heat* between *two fluids* that are at different temperatures while keeping them from mixing with each other. Heat exchangers are commonly used in practice in a wide range of applications, from heating and air-conditioning systems in a household, to chemical processing and power production in large plants. Heat exchangers differ from mixing chambers in that they do not allow the two fluids involved to mix. In a car radiator, for example, heat is transferred from the hot water flowing through the radiator tubes to the air flowing through the closely spaced thin plates outside attached to the tubes.

Heat transfer in a heat exchanger usually involves *convection* in each fluid and *conduction* through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an *overall heat transfer coefficient U* that accounts for the contribution of all these effects on heat transfer. The rate of heat transfer between the two fluids at a location in a heat exchanger depends on the magnitude of the temperature difference at that location, which varies along the heat exchanger. In the analysis of heat exchangers, it is usually convenient to work with the *logarithmic mean temperature difference LMTD,* which is an equivalent mean temperature difference between the two fluids for the entire heat exchanger.

Heat exchangers are manufactured in a variety of types, and thus we start this chapter with the *classification* of heat exchangers. We then discuss the determination of the overall heat transfer coefficient in heat exchangers, and the logarithmic mean temperature difference LMTD for some configurations. We then introduce the *correction factor F* to account for the deviation of the mean temperature difference from the LMTD in complex configurations. Next we discuss the effectiveness-NTU method, which enables us to analyze heat exchangers when the outlet temperatures of the fluids are not known. Finally, we discuss the selection of heat exchangers.

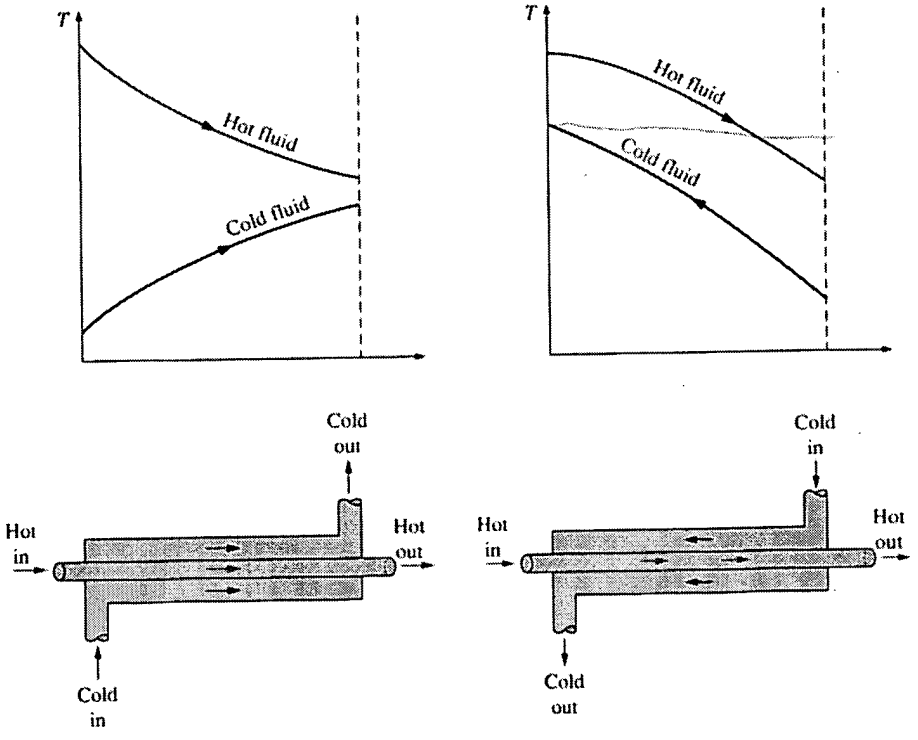
10-1 ■ TYPES OF HEAT EXCHANGERS

Different heat transfer applications require different types of hardware and different configurations of heat transfer equipment. The attempt to match the heat transfer hardware to the heat transfer requirements within the specified constraints has resulted in numerous types of innovative heat exchanger designs.

The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Figure 10-1, called the **double-pipe heat exchanger**. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: in **parallel flow**, both the hot and cold fluids enter the heat exchanger at the same end and move in the *same* direction. In **counter flow**, on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in *opposite* directions.

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the **compact heat exchanger**. The ratio of the heat transfer surface area of a heat exchanger to its

FIGURE 10-1
Different flow regimes and associated temperature profiles in a double-pipe heat exchanger.



(*a*) Parallel flow

(*b*) Counter flow

called the *area density* β. A heat exchanger with $\beta > 700$ m^2/m^3 ft^2/ft^3) is classified as being compact. Examples of compact heat ers are car radiators ($\beta \approx 1000$ m^2/m^3), glass ceramic gas turbine gers ($\beta \approx 6000$ m^2/m^3), the regenerator of a Stirling engine m^2/m^3), and the human lung ($\beta \approx 20{,}000$ m^2/m^3). Compact heat enable us to achieve high heat transfer rates between two fluids in lume, and they are commonly used in applications with strict limi- the weight and volume of heat exchangers (Fig. 10-2).

large surface area in compact heat exchangers is obtained by closely spaced *thin plate* or *corrugated fins* to the walls separating fluids. Compact heat exchangers are commonly used in gas-to-gas -liquid (or liquid-to-gas) heat exchangers to counteract the low heat efficient associated with gas flow with increased surface area. In a or, which is a water-to-air compact heat exchanger, for example, it pose that fins are attached to the air side of the tube surface.

compact heat exchangers, the two fluids usually move *perpendicular* ther, and such flow configuration is called **cross-flow.** The cross- ther classified as *unmixed* and *mixed flow*, depending on the flow tion, as shown in Figure 10-3. In (*a*) the cross-flow is said to be since the plate fins force the fluid to flow through a particular interfin nd prevent it from moving in the transverse direction (i.e., parallel ig. 10-3). The cross-flow in (*b*) is said to be *mixed* since the fluid now is move in the transverse direction. Both fluids are unmixed in a car The presence of mixing in the fluid can have a significant effect on transfer characteristics of the heat exchanger.

ps the most common type of heat exchanger in industrial applica- the **shell-and-tube** heat exchanger, shown in Figure 10-4. Shell-and-

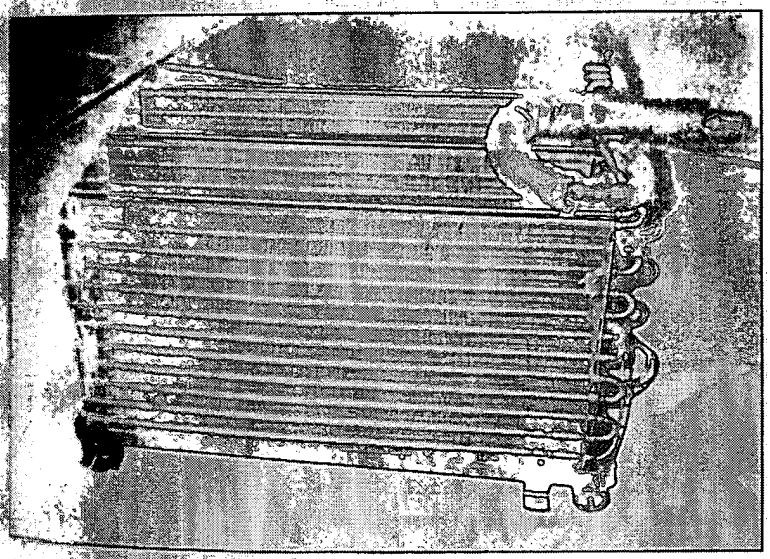


FIGURE 10-2
A gas-to-liquid compact heat exchanger for a residential air-conditioning system.

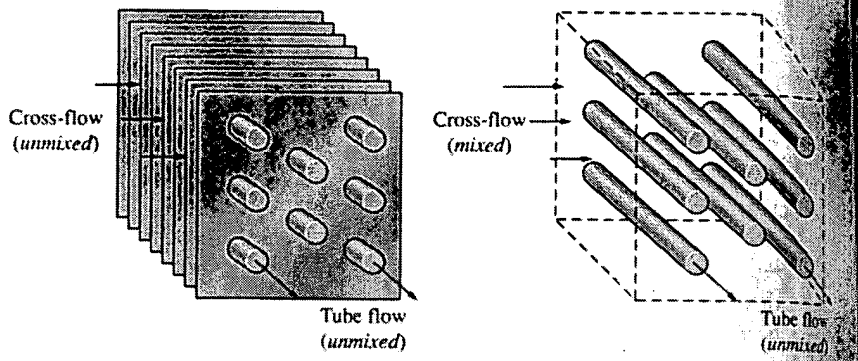


FIGURE 10-3
Different flow configurations in cross-flow heat exchangers.

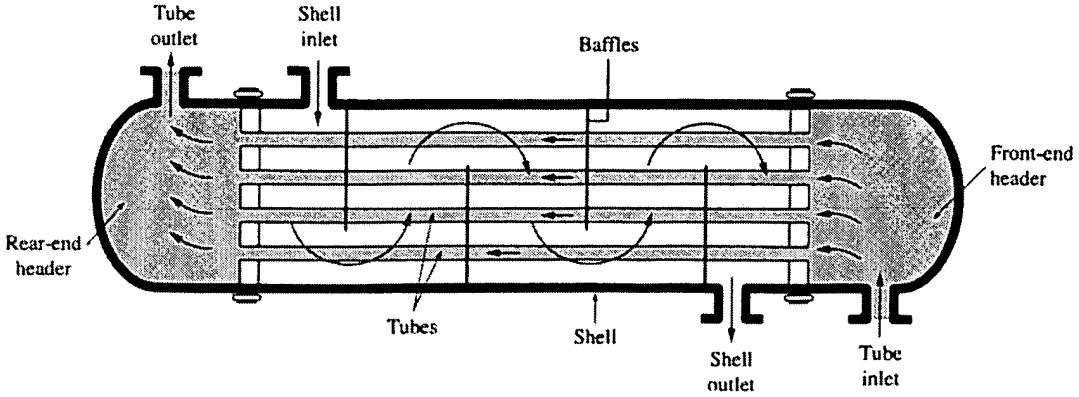
(*a*) Both fluids unmixed (*b*) One fluid mixed, one fluid unmixed

tube heat exchangers contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. *Baffles* are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes. Despite their widespread use, shell-and-tube heat exchangers are not suitable for use in automotive, aircraft, and marine applications because of their relatively large size and weight. Note that the tubes in a shell-and-tube heat exchanger open to some large flow areas called *headers* at both ends of the shell, where the tube-side fluid accumulates before entering the tubes and after leaving them.

Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U-turn in the shell, for example, are called *one-shell pass and two-tube pass* heat exchangers. Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a *two-shell pass and four-tube pass* heat exchanger (Fig. 10-5).

An innovative type of heat exchanger that has found widespread use is the **plate and frame** (or just plate) heat exchanger, which consists of a series

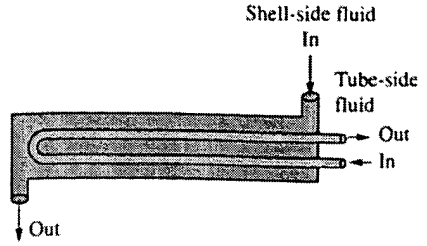
FIGURE 10-4
The schematic of a shell-and-tube heat exchanger (one-shell pass and one-tube pass).



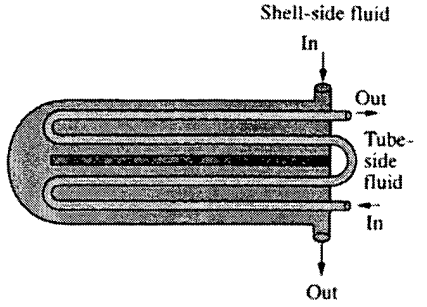
of plates with corrogated flat flow passages (Fig. 10-6). The hot and cold fluids flow in alternate passages, and thus each cold fluid stream is surrounded by two hot fluid streams, resulting in very effective heat transfer. Also, plate heat exchangers can grow with increasing demand for heat transfer by simply mounting more plates. They are well suited for liquid-to-liquid heat exchange applications, provided that the hot and cold fluid streams are at about the same pressure.

Another type of heat exchanger that involves the alternate passage of the hot and cold fluid streams through the same flow area is the **regenerative** heat exchanger. The *static*-type regenerative heat exchanger is basically a porous mass that has a large heat storage capacity, such as a ceramic wire mesh. Hot and cold fluids flow through this porous mass alternatively. Heat is transferred from the hot fluid to the matrix of the regenerator during the flow of the hot fluid, and from the matrix to the cold fluid during the flow of the cold fluid. Thus, the matrix serves as a temporary heat storage medium.

The *dynamic*-type regenerator involves a rotating drum and continuous flow of the hot and cold fluid through different portions of the drum so that



(*a*) One-shell pass and two-tube passes



(*b*) Two-shell passes and four-tube passes

FIGURE 10-5

Multipass flow arrangements in shell-and-tube heat exchangers.

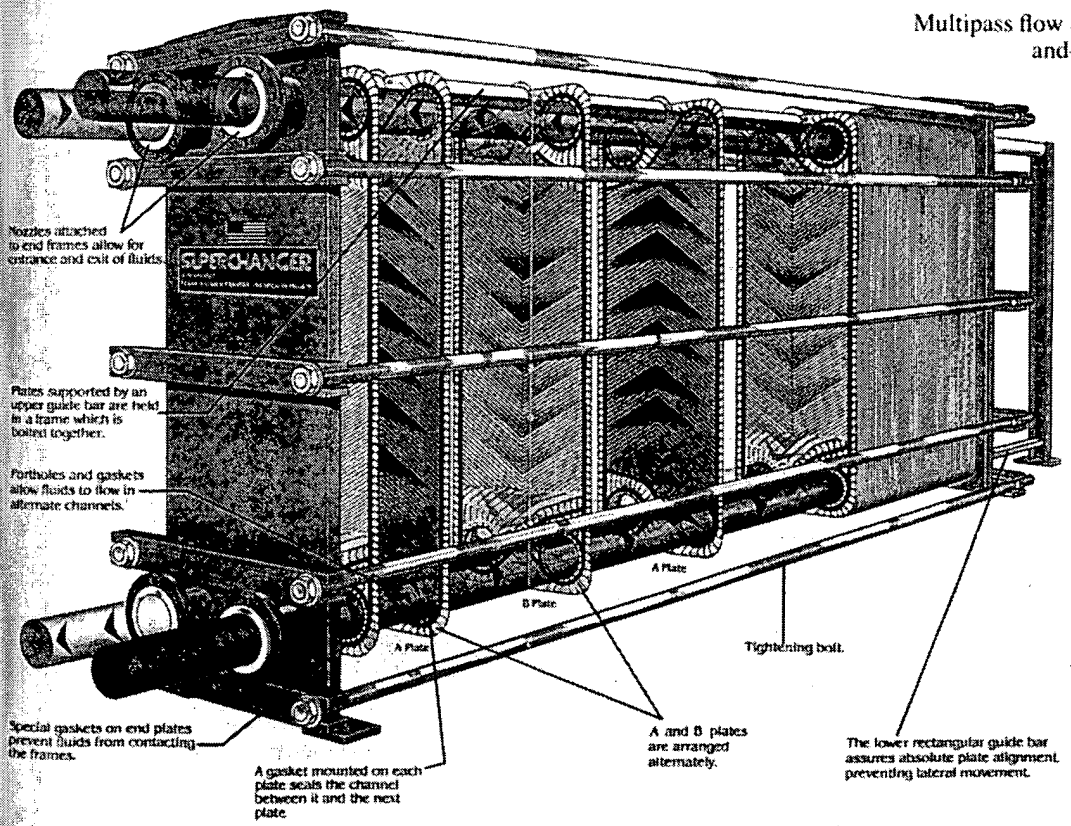


FIGURE 10-6

A plate-and-frame liquid-to-liquid heat exchanger (courtesy of Trante Corp.).

CHAPTER 10
Heat Exchangers

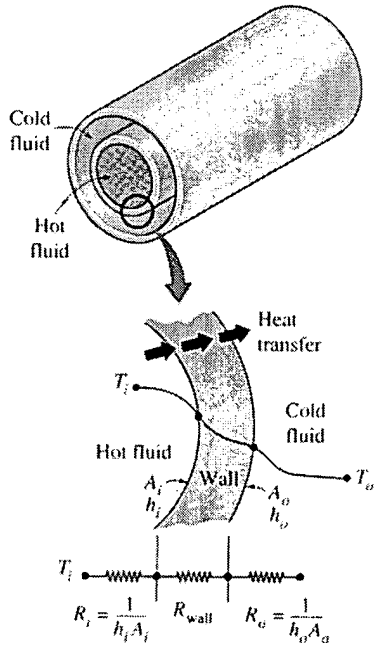


FIGURE 10-7
Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

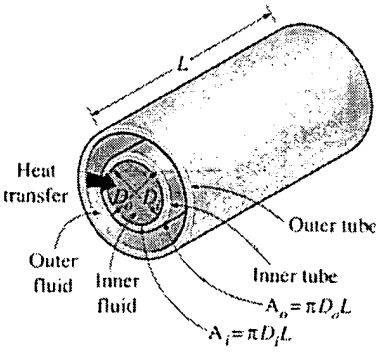


FIGURE 10-8
The two heat transfer surface areas associated with a double-pipe heat exchanger (for thin tubes, $D_i \approx D_o$ and thus $A_i \approx A_o$).

any portion of the drum passes periodically through the hot stream, storing heat, and then through the cold stream, *rejecting* this *stored* heat. Again the drum serves as the medium to transport the heat from the hot to the cold fluid stream.

Heat exchangers are often given specific names to reflect the specific application for which they are used. For example, a *condenser* is a heat exchanger in which one of the fluids gives up heat and condenses as it flows through the heat exchanger. A *boiler* is another heat exchanger in which one of the fluids absorbs heat and vaporizes. A *space radiator* is a heat exchanger that transfers heat from the hot fluid to the surrounding space by radiation.

10-2 ■ THE OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by *convection*, through the wall by *conduction*, and from the wall to the cold fluid again by *convection*. Any radiation effects are usually included in the convection heat transfer coefficients.

The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in Figure 10-7. Here the subscripts i and o represent the inner and outer surfaces of the inner tube. For a double-pipe heat exchanger, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$, and the *thermal resistance* of the tube wall in this case is

$$R_{\text{wall}} = \frac{\ln (D_o/D_i)}{2\pi kL} \tag{10-1}$$

where k is the thermal conductivity of the wall material and L is the length of the tube. Then the *total thermal resistance* becomes

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln (D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o} \tag{10-2}$$

The A_i is the area of the *inner surface* of the wall that separates the two fluids, and A_o is the area of the outer surface of the wall. In other words, A_i and A_o are surface areas of the separating wall wetted by the inner and the outer fluids, respectively. When one fluid flows inside a circular tube and the other outside of it, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$ (Fig. 10-8).

In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance R, as we discussed in Chapter 3, and to express the rate of heat transfer between the two fluids as

$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T \tag{10-3}$$

where U is the **overall heat transfer coefficient**, whose unit is W/m² · °C which is identical to the unit of the ordinary convection coefficient h. Given celing ΔT. Equation 10-3 reduces to

$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{wall} + \frac{1}{h_o A_o} \quad (10\text{-}4)$$

Perhaps you are wondering why we have *two* overall heat transfer coefficients U_i and U_o for a heat exchanger. The reason is that every heat exchanger has two heat transfer surface areas A_i and A_o, which, in general, are not equal to each other.

Note that $U_i A_i = U_o A_o$, but $U_i \neq U_o$ unless $A_i = A_o$. Therefore, the overall heat transfer coefficient U of a heat exchanger is meaningless unless the area on which it is based is specified. This is especially the case when one side of the tube wall is finned and the other side is not, since the surface area of the finned side is several times that of the unfinned side.

When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, as is usually the case, the thermal resistance of the tube is negligible ($R_{wall} \approx 0$) and the inner and outer surfaces of the tube are almost identical ($A_i \approx A_o \approx A$). Then Equation 10-4 for the overall heat transfer coefficient simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \quad (10\text{-}5)$$

where $U \approx U_i \approx U_o$. The individual convection heat transfer coefficients inside and outside the tube, h_i and h_o, are determined using the convection relations discussed in earlier chapters.

The overall heat transfer coefficient U in Equation 10-5 is dominated by the *smaller* convection coefficient, since the inverse of a large number is small. When one of the convection coefficients is *much smaller* than the other (say, $h_i \ll h_o$), we have $1/h_i \gg 1/h_o$, and thus $U \approx h_i$. Therefore, the smaller heat transfer coefficient creates a *bottleneck* on the path of heat flow and seriously impedes heat transfer. This situation arises frequently when one of the fluids is a gas and the other is a liquid. In such cases, fins are commonly used on the gas side to enhance the product UA and thus the heat transfer on that side.

Representative values of the overall heat transfer coefficient U are given in Table 10-1. Note that the overall heat transfer coefficient ranges from about 10 W/m$^2 \cdot$ °C for gas-to-gas heat exchangers to about 10,000 W/m$^2 \cdot$ °C for heat exchangers that involve phase changes. This is not surprising, since gases have very low thermal conductivities, and phase-change processes involve very high heat transfer coefficients.

When the tube is *finned* on one side to enhance heat transfer, the total heat transfer surface area on the fined side becomes

$$A = A_{total} = A_{fin} + A_{unfinned} \quad (10\text{-}6)$$

where A_{fin} is the surface area of the fins and $A_{unfinned}$ is the area of unfinned portion of the tube surface. For short fins of high thermal conductivity, we can use this total area in the convection resistance relation $R_{conv} = 1/hA$ since the fins in this case will be very nearly isothermal. Otherwise, we should determine the effective surface are A from

TABLE 10-1

Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	U, W/m² · °C*
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60†
	400–850†
Steam-to-air in finned tubes (steam in tubes)	30–300†
	400–4000‡

*Multiply the listed values by 0.176 to convert them to Btu/h · ft² · °F.
†Based on air-side surface area.
‡Based on water- or steam-side surface area.

$$A = A_{\text{unfinned}} + \eta_{\text{fin}} A_{\text{fin}} \quad (10\text{-}7)$$

where η_{fin} is the fin efficiency. This way, the temperature drop along the fins is accounted for. Note that $\eta_{\text{fin}} = 1$ for isothermal fins, and thus Equation 10-7 reduces to Equation 10-6 in that case.

Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of *deposits* on heat transfer surfaces. The layer of deposits represents *additional resistance* to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a **fouling factor** R_f, which is a measure of the *thermal resistance* introduced by fouling.

The most common type of fouling is the *precipitation* of solid deposits in a fluid on the heat transfer surfaces. You can observe this type of fouling even in your house. If you check the inner surfaces of your teapot after prolonged use, you will probably notice a layer of calcium-based deposits on the surfaces at which boiling occurs. This is especially the case in areas where the water is hard. The scales of such deposits come off by scratching, and the surfaces can be cleaned of such deposits by chemical treatment. Now imagine

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FIGURE 10-9
Precipitation fouling of ash particles on superheater tubes (from *Steam, Its Generation, and Use*, Babcock and Wilcox Co., 1978).

those mineral deposits forming on the inner surfaces of fine tubes in a heat exchanger (Fig. 10-9) and the detrimental effect it may have on the flow passage area and the heat transfer. To avoid this potential problem, water in power and process plants is extensively treated and its solid contents are removed before it is allowed to circulate through the system. The solid ash particles in the flue gases accumulating on the surfaces of air preheaters create similar problems.

Another form of fouling, which is common in the chemical process industry, is *corrosion* and other *chemical fouling*. In this case, the surfaces are fouled by the accumulation of the products of chemical reactions on the surfaces. This form of fouling can be avoided by coating metal pipes with glass or using plastic pipes instead of metal ones. Heat exchangers may also be fouled by the growth of algae in warm fluids. This type of fouling is called *biological fouling* and can be prevented by chemical treatment.

In applications where it is likely to occur, fouling should be considered in the design and selection of heat exchangers. In such applications, it may be necessary to select a larger and thus more expensive heat exchanger to ensure that it meets the design heat transfer requirements even after fouling occurs. The periodic cleaning of heat exchangers and the resulting down time are additional penalties associated with fouling.

The fouling factor is obviously zero for a new heat exchanger and increases with time as the solid deposits build up on the heat exchanger surface. The fouling factor depends on the *operating temperature* and the *velocity* of the fluids, as well as the length of service. Fouling increases with *increasing temperature* and *decreasing velocity*.

The overall heat transfer coefficient relation given above is valid for clean surfaces and needs to be modified to account for the effects of fouling on both the inner and the outer surfaces of the tube. For an unfinned shell-and-tube heat exchanger, it can be expressed as

CHAPTER 10
Heat Exchangers

$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln (D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o} \quad (10\text{-}8)$$

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of inner and outer surfaces, and R_{fi} and R_{fo} are the fouling factors at those surfaces.

Representative values of fouling factors are given in Table 10-2. More comprehensive tables of fouling factors are available in handbooks. As you would expect, considerable uncertainty exists in these values, and they should be used as a guide in the selection and evaluation of heat exchangers to account for the effects of anticipated fouling on heat transfer. Note that most fouling factors in the table are of the order of 10^{-4} m² · °C/W, which is equivalent to the thermal resistance of a 0.2-mm-thick limestone layer (k = 2.9 W/m · °C) per unit surface area. Therefore, in the absence of specific data, we can assume the surfaces to be coated with 0.2 mm of limestone as a starting point to account for the effects of fouling.

TABLE 10-2

Representative fouling factors (thermal resistance due to fouling for a unit surface area)

Fluid	R_f, m² · °C/W
Distilled water, sea water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

Source: Tubular Exchange Manufacturers Association.

EXAMPLE 10-1 Overall Heat Transfer Coefficient of a Heat Exchanger

Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tube has a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.

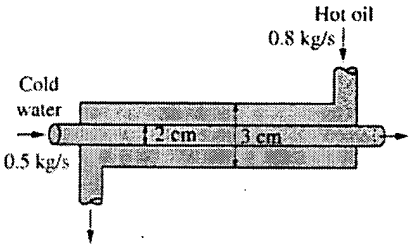


FIGURE 10-10
Schematic for Example 10-1.

Solution Hot oil is cooled by water in a double-tube counterflow heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the oil and water flow are fully developed. **3** Proberties of the oil and water are constant.

Properties The properties of water at 45°C are (Table A-9)

$$\rho = 990 \text{ kg/m}^3 \qquad \text{Pr} = 3.91$$
$$k = 0.637 \text{ W/m} \cdot \text{°C} \qquad \nu = \mu/\rho = 0.602 \times 10^{-6} \text{ m}^2/\text{s}$$

The properties of oil at 80°C are (Table A-10).

$$\rho = 852 \text{ kg/m}^3 \qquad \text{Pr} = 490$$
$$k = 0.138 \text{ W/m} \cdot \text{°C} \qquad \nu = 37.5 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis The schematic of the heat exchanger is given in Figure 10-10. The overall heat transfer coefficient U can be determined from Equation 10-5:

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where h_i and h_o are the convection heat transfer coefficients inside and outside the tube, respectively, which are to be determined using the forced convection relations discussed in Chapter 6.

The hydraulic diameter for a circular tube is the diameter of the tube itself, $D_h = D = 0.02$ m. The average velocity of water in the tube and the Reynolds number are

$$V_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{1}{4}\pi D^2)} = \frac{0.5 \text{ kg/s}}{(990 \text{ kg/m}^3)[\frac{1}{4}\pi(0.02 \text{ m})^2]} = 1.61 \text{ m/s}$$

$$\text{Re} = \frac{V_m D_h}{\nu} = \frac{(1.61 \text{ m/s})(0.02 \text{ m})}{0.602 \times 10^{-6} \text{ m}^2/\text{s}} = 53{,}490$$

greater than 4000. Therefore, the flow of water is turbulent. Assuming to be fully developed, the Nusselt number can be determined from

$$\text{Nu} = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8}\text{Pr}^{0.4} = 0.023(53{,}490)^{0.8}(3.91)^{0.4} = 240.6$$

$$h = \frac{k}{D_h}\text{Nu} = \frac{0.637 \text{ W/m} \cdot ^\circ\text{C}}{0.02 \text{ m}}(240.6) = 7663 \text{ W/m}^2 \cdot ^\circ\text{C}$$

we repeat the analysis above for oil. The properties of oil at 80°C are

$$\rho = 852 \text{ kg/m}^3 \qquad \nu = 37.5 \times 10^{-6} \text{ m}^2/\text{s}$$
$$k = 0.138 \text{ W/m} \cdot ^\circ\text{C} \qquad \text{Pr} = 490$$

draulic diameter for the annular space is

$$D_h = D_o - D_i = 0.03 - 0.02 = 0.01\text{m}$$

rage velocity and the Reynolds number in this case are

$$V_m = \frac{\dot{m}}{\rho[\frac{1}{4}\pi(D_o^2 - D_i^2)]} = \frac{0.8 \text{ kg/s}}{(852 \text{ kg/m}^3)[\frac{1}{4}\pi(0.03^2 - 0.02^2)] \text{ m}^2} = 2.39 \text{ m/s}$$

$$\text{Re} = \frac{V_m D_h}{\nu} = \frac{(2.39 \text{ m/s})(0.01 \text{ m})}{37.5 \times 10^{-6} \text{ m}^2/\text{s}} = 637$$

less than 4000. Therefore, the flow of oil is laminar. Assuming fully devel- ow, the Nusselt number on the tube side of the annular space Nu_i corre- g to $D_i/D_o = 0.02/0.03 = 0.667$ can be determined from Table 10-3 by ation to be

$$\text{Nu} = 5.45$$

$$h_o = \frac{k}{D_h}\text{Nu} = \frac{0.138 \text{ W/m} \cdot ^\circ\text{C}}{0.01 \text{ m}}(5.45) = 75.2 \text{ W/m}^2 \cdot ^\circ\text{C}$$

e overall heat transfer coefficient for this heat exchanger becomes

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7663 \text{ W/m}^2 \cdot ^\circ\text{C}} + \frac{1}{75.2 \text{ W/m}^2 \cdot ^\circ\text{C}}} = \mathbf{74.5 \text{ W/m}^2 \cdot ^\circ\text{C}}$$

on Note that $U \approx h_o$ in this case, since $h_i \gg h_o$. This confirms our earlier that the overall heat transfer coefficient in a heat exchanger is

TABLE 10-3

Nusselt number for fully developed laminar flow in a circular annulus with one surface insulated and the other isothermal

D_i/D_o	Nu_i	Nu_o
0.00	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

Source: Kays and Perkins, Ref. 8.

dominated by the smaller heat transfer coefficient when the difference between the two values is large.

To improve the overall heat transfer coefficient and thus the heat transfer in this heat exchanger, we must use some enhancement techniques on the oil side, such as a finned surface.

EXAMPLE 10-2 Effect of Fouling on the Overall Heat Transfer Coefficient

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel (k = 15.1 W/m · °C) inner tube of inner diameter D_i = 1.5 cm and outer diameter D_o = 1.9 cm and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be h_i = 800 W/m² · °C on the inner surface of the tube and h_o = 1200 W/m² · °C on the outer surface. For a fouling factor of $R_{f,i}$ = 0.0004 m² · °C/W on the tube side and $R_{f,o}$ = 0.0001 m² · °C/W on the shell side, determine (*a*) the thermal resistance of the heat exchanger per unit length and (*b*) the overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.

Solution The heat transfer coefficients and the fouling factors on tube and shell side of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis (*a*) The schematic of the heat exchanger is given in Figure 10-11. The thermal resistance for an unfinned shell-and-tube heat exchanger with fouling on both heat transfer surfaces is given by Equation 10-8 as

$$R = \frac{1}{UA} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where

$$A_i = \pi D_iL = \pi(0.015\text{ m})(1\text{ m}) = 0.0471\text{ m}^2$$
$$A_o = \pi D_oL = \pi(0.019\text{ m})(1\text{ m}) = 0.0597\text{ m}^2$$

Substituting, the total thermal resistance is determined to be

$$R = \frac{1}{(800\text{ W/m}^2\cdot\text{°C})(0.0471\text{ m}^2)} + \frac{0.0004\text{ m}^2\cdot\text{°C/W}}{0.0471\text{ m}^2} + \frac{\ln(0.019/0.015)}{2\pi(15.1\text{ W/m}\cdot\text{°C})(1\text{ m})} + \frac{0.0001\text{ m}^2\cdot\text{°C/W}}{0.0597\text{ m}^2} + \frac{1}{(1200\text{ W/m}^2\cdot\text{°C})(0.0597\text{ m}^2)}$$
$$= (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396)\text{°C/W}$$
$$= \mathbf{0.0532\text{°C/W}}$$

Discussion Note that about 19 percent of the total thermal resistance in this case is due to fouling and about 5 percent of it is due to the steel tube separating the two fluids. The rest (76 percent) is due to the convection resistances on the two sides of the inner tube.

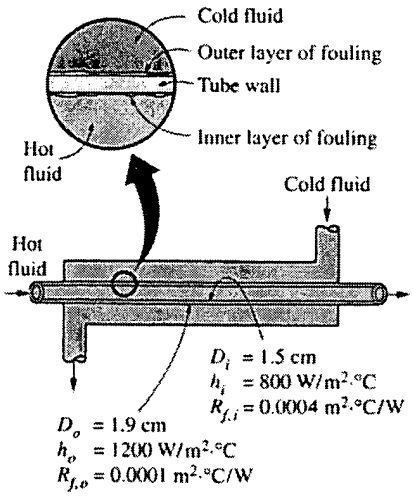


FIGURE 10-11
Schematic for Example 10-2.

...ing the total thermal resistance and the heat transfer surface areas, the ...eat transfer coefficient based on the inner and outer surfaces of the tube ...rmined again from Equation 10-8 to be

$$U_i = \frac{1}{R A_i} = \frac{1}{(0.0532\ ^\circ\text{C/W})(0.0471\ \text{m}^2)} = 399.1\ \text{W/m}^2 \cdot ^\circ\text{C}$$

$$U_o = \frac{1}{R A_o} = \frac{1}{(0.0532\ ^\circ\text{C/W})(0.0597\ \text{m}^2)} = 314.9\ \text{W/m}^2 \cdot ^\circ\text{C}$$

...sion. Note that the two overall heat transfer coefficients differ significantly ...percent) in this case because of the considerable difference between the ...transfer surface areas on the inner and the outer sides of the tube. For tubes ...gible thickness, the difference between the two overall heat transfer co- ...ents would be negligible.

10-3 ANALYSIS OF HEAT EXCHANGERS

...exchangers are commonly used in practice, and an engineer often finds ...self or herself in a position to *select a heat exchanger* that will achieve a ...d *temperature change* in a fluid stream of known mass flow rate, or to ...the *outlet temperatures* of the hot and cold fluid streams in a *specified* ...exchanger.

In the following sections, we will discuss the two methods used in the ...ysis of heat exchangers. Of these, the *log mean temperature difference* ...(LMTD) method is best suited for the first task and the *effectiveness-NTU* ...od for the second task stated above. But first we present some general ...derations.

Heat exchangers usually operate for long periods of time with no change ...their operating conditions. Therefore, they can be modeled as *steady-flow* ... As such, the mass flow rate of each fluid remains constant, and the ...d properties such as temperature and velocity at any inlet or outlet remain ...e same. Also, the fluid streams experience little or no change in their ...ocities and elevations, and thus the *kinetic* and *potential energy changes* ...e negligible. The *specific heat* of a fluid, in general, changes with tempera- ...But in a specified temperature range, it can be treated as a constant at ...average value with little loss in accuracy. *Axial heat conduction* along ...tube is usually insignificant and can be considered negligible. Finally, the ...r surface of the heat exchanger is assumed to be *perfectly insulated*, so ...here is no heat loss to the surrounding medium, and any heat transfer ...between the two fluids only.

...idealizations stated above are closely approximated in practice, and ...greatly simplify the analysis of a heat exchanger with little sacrifice of ...cy. Therefore, they are commonly used. Under these assumptions, the ...of *thermodynamics* requires that the rate of heat transfer from ...fluid be equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) \tag{10-9}$$

and

$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,\text{in}} - T_{h,\text{out}}) \tag{10-10}$$

where the subscripts c and h stand for *cold* and *hot* fluids, respectively, and

$\dot{m}_c, \dot{m}_h$ = mass flow rates
C_{pc}, C_{ph} = specific heats
$T_{c,\text{out}}, T_{h,\text{out}}$ = outlet temperatures
$T_{c,\text{in}}, T_{h,\text{in}}$ = inlet temperatures

Note that the heat transfer rate $\dot{Q}$ is taken to be a positive quantity, and its direction is understood to be from the hot fluid to the cold one in accordance with the second law of thermodynamics.

In heat exchanger analysis, it is often convenient to combine the product of the *mass flow rate* and the *specific heat* of a fluid into a single quantity. This quantity is called the **heat capacity rate** and is defined as

$$C = \dot{m}C_p \tag{10-11}$$

The heat capacity rate of a fluid stream represents the rate of heat transfer needed to change the temperature of the fluid stream by 1°C as it flows through a heat exchanger. Note that in a heat exchanger, the fluid with a *large* heat capacity rate will experience a *small* temperature change, and the fluid with a *small* heat capacity rate will experience a *large* temperature change. Therefore, *doubling* the mass flow rate of a fluid while leaving everything else unchanged will *halve* the temperature change of that fluid.

With the definition of the heat capacity rate above, Equations 10-9 and 10-10 can also be expressed as

$$\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) \tag{10-12}$$

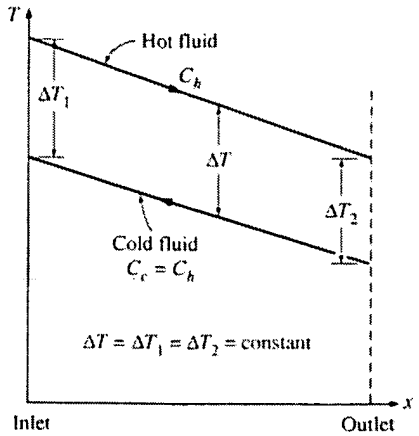
and

$$\dot{Q} = C_h(T_{h,\text{in}} - T_{h,\text{out}}) \tag{10-13}$$

That is, the heat transfer rate in a heat exchanger is equal to the heat capacity rate of either fluid multiplied by the temperature change of that fluid. Note that *the only time the temperature rise of a cold fluid is equal to the temperature drop of the hot fluid is when the heat capacity rates of the two fluids are equal to each other* (Fig. 10-12).

Two special types of heat exchangers commonly used in practice are *condensers* and *boilers*. One of the fluids in a condenser or a boiler undergoes a phase-change process, and the rate of heat transfer is expressed as

FIGURE 10-12
Two fluids that have the same mass flow rate and the same specific heat experience the same temperature change in a well-insulated heat exchanger.



$$\dot{Q} = \dot{m} h_{fg} \tag{10-14}$$

where $\dot{m}$ is the rate of evaporation or condensation of the fluid and h_{fg} is the enthalpy of vaporization of the fluid at the specified temperature or pressure.

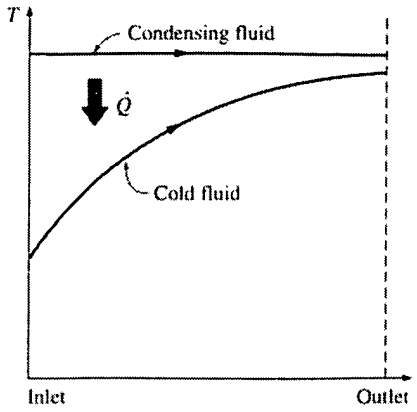
An ordinary fluid absorbs or releases a large amount of heat essentially at constant temperature during a phase-change process, as shown in Figure 10-13. The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero. That is, $C = \dot{m}C_p \to \infty$ when $\Delta T \to 0$, so that the heat transfer rate $\dot{Q} = \dot{m}C_p \Delta T$ is a finite quantity. Therefore, in heat exchanger analysis, a condensing or boiling fluid is conveniently modeled as a fluid whose heat capacity rate is *infinity*.

The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to Newton's law of cooling as

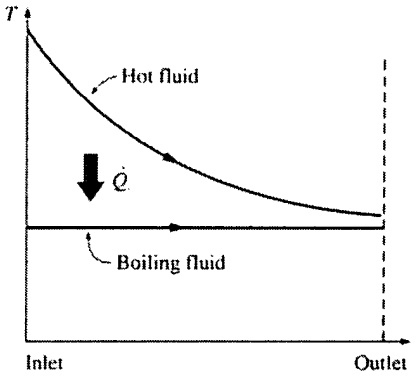
$$\dot{Q} = UA\,\Delta T_m \tag{10-15}$$

where U is the overall heat transfer coefficient, A is the heat transfer area, and ΔT_m is an appropriate average temperature difference between the two fluids. Here the surface area A can be determined precisely using the dimensions of the heat exchanger. However, the overall heat transfer coefficient U and the temperature difference ΔT between the hot and cold fluids, in general, are not constant and vary along the heat exchanger.

The average value of the overall heat transfer coefficient can be determined as described in the preceding section by using the average convection coefficients for each fluid. It turns out that the appropriate form of the mean temperature difference between the two fluids is *logarithmic* in nature, and its determination is presented in the next section.



(*a*) Condenser ($C_h \to \infty$)



(*b*) Boiler ($C_c \to \infty$)

FIGURE 10-13
Variation of fluid temperatures in a heat exchanger when one of the fluids condenses or boils.

10-4 ■ THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

Earlier, we mentioned that the temperature difference between the hot and cold fluids varies along the heat exchanger, and it is convenient to have a *mean temperature difference* ΔT_m for use in the relation

$$\dot{Q} = UA\,\Delta T_m$$

In order to develop a relation for the equivalent average temperature difference between the two fluids, consider the *parallel-flow double-pipe* heat exchanger shown in Figure 10-14. Note that the temperature difference ΔT between the hot and cold fluids is large at the inlet of the heat exchanger but decreases exponentially toward the outlet. As you would expect, the temperature of the hot fluid decreases and the temperature of the cold fluid increases along the heat exchanger, but the temperature of the cold fluid can never exceed that of the hot fluid no matter how long the heat exchanger is.

Assuming the outer surface of the heat exchanger to be well insulated so that any heat transfer occurs between the two fluids, and disregarding any changes in kinetic and potential energy, an energy balance on each fluid in a

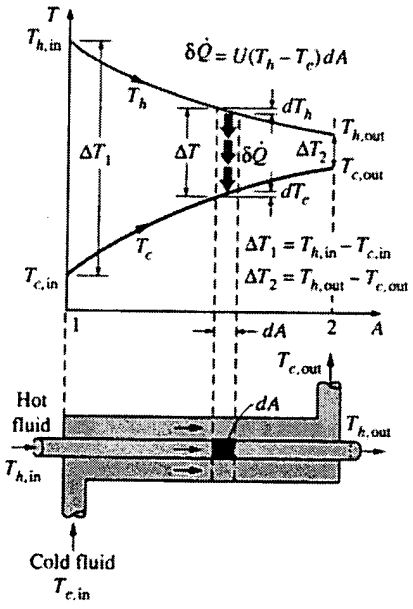


FIGURE 10-14
Variation of the fluid temperatures in a parallel-flow double-pipe heat exchanger.

differential section of the heat exchanger can be expressed as

$$\delta\dot{Q} = -\dot{m}_h C_{ph} dT_h \tag{10-16}$$

and

$$\delta\dot{Q} = \dot{m}_c C_{pc} dT_c \tag{10-17}$$

That is, the rate of heat loss from the hot fluid at any section of a heat exchanger is equal to the rate of heat gain by the cold fluid in that section. The temperature change of the hot fluid is a *negative* quantity, and so a *negative sign* is added to Equation 10-16 to make the heat transfer rate $\dot{Q}$ a positive quantity. Solving the equations above for dT_h and dT_c gives

$$dT_h = -\frac{\delta\dot{Q}}{\dot{m}_h C_{ph}} \tag{10-18}$$

and

$$dT_c = \frac{\delta\dot{Q}}{\dot{m}_c C_{pc}} \tag{10-19}$$

Taking their difference, we get

$$dT_h - dT_c = d(T_h - T_c) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{10-20}$$

The rate of heat transfer in the differential section of the heat exchanger can also be expressed as

$$\delta\dot{Q} = U(T_h - T_c)dA \tag{10-21}$$

Substituting this equation into Equation 10-20 and rearranging gives

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\,dA\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{10-22}$$

Integrating from the inlet of the heat exchanger to its outlet, we obtain

$$\ln\frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}} = -UA\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{10-23}$$

Finally, solving Equations 10-9 and 10-10 for $\dot{m}_c C_{pc}$ and $\dot{m}_h C_{ph}$ and substituting into Equation 10-23 gives, after some rearrangement,

$$\dot{Q} = UA\,\Delta T_{lm} \tag{10-24}$$

where

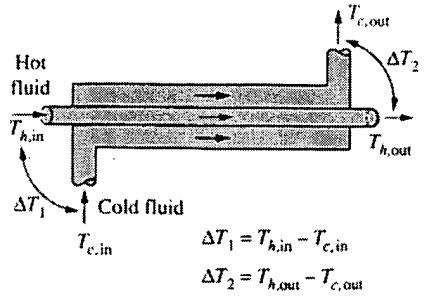
$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{10-25}$$

is the **log mean temperature difference**, which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature difference between the two fluids

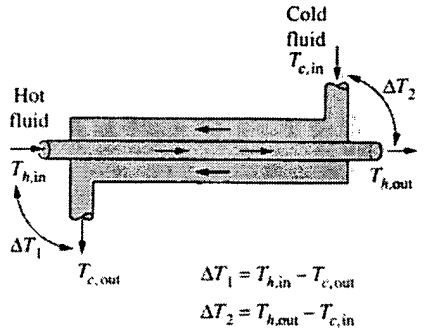
at the two ends (inlet and outlet) of the heat exchanger. It makes no difference which end of the heat exchanger is designated as the *inlet* or the *outlet* (Fig. 10-15).

The temperature difference between the two fluids decreases from ΔT_1 at the inlet to ΔT_2 at the outlet. Thus, it is tempting to use the arithmetic mean temperature $\Delta T_{am} = \frac{1}{2}(\Delta T_1 + \Delta T_2)$ as the average temperature difference. The logarithmic mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluids along the heat exchanger and is an *exact* representation of the *average temperature difference* between the hot and cold fluids. It truly reflects the exponential decay of the local temperature difference.

Note that ΔT_{lm} is always less than ΔT_{am}. Therefore, using ΔT_{am} in calculations instead of ΔT_{lm} will overestimate the rate of heat transfer in a heat exchanger between the two fluids. When ΔT_1 differs from ΔT_2 by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_1 differs from ΔT_2 by greater amounts. Therefore, we should always use the *logarithmic mean temperature difference* when determining the rate of heat transfer in a heat exchanger.



(*a*) Parallel-flow heat exchangers



(*b*) Counter-flow heat exchangers

FIGURE 10-15

The ΔT_1 and ΔT_2 expressions in parallel-flow and counter-flow heat exchangers.

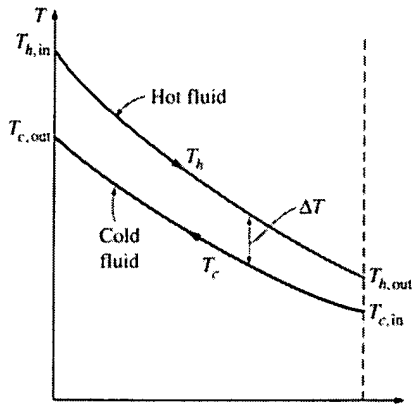
Counter-Flow Heat Exchangers

The variation of temperatures of hot and cold fluids in a counter-flow heat exchanger is given in Figure 10-16. Note that the hot and cold fluids enter the heat exchanger from opposite ends, and the outlet temperature of the *cold fluid* in this case may exceed the outlet temperature of the *hot fluid*. In the limiting case, the cold fluid will be heated to the inlet temperature of the hot fluid. However, the outlet temperature of the cold fluid can *never* exceed the inlet temperature of the hot fluid, since this would be a violation of the second law of thermodynamics.

The relation above for the log mean temperature difference is developed using a parallel-flow heat exchanger, but we can show by repeating the analysis above for a counter-flow heat exchanger that is also applicable to counter-flow heat exchangers. But this time, ΔT_1 and ΔT_2 are expressed as shown in Figure 10-15.

For specified inlet and outlet temperatures, the log mean temperature difference for a *counter-flow* heat exchanger is always *greater* than that for a parallel-flow heat exchanger. That is, $\Delta T_{lm,CF} > \Delta T_{lm,PF}$, and thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counter-flow heat exchanger. Therefore, it is common practice to use counter-flow arrangements in heat exchangers.

In a counter-flow heat exchanger, the temperature difference between the hot and the cold fluids will remain constant along the heat exchanger when the *heat capacity rates* of the two fluids are *equal* (that is, ΔT = constant when $C_h = C_c$ or $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$). Then we have $\Delta T_1 = \Delta T_2$, and the log mean temperature difference relation above gives $\Delta T_{lm} = \frac{0}{0}$, which is indeterminate. It can be shown by the application of l'Hôpital's rule that in this case we have $\Delta T_{lm} = \Delta T_1 = \Delta T_2$, as expected.



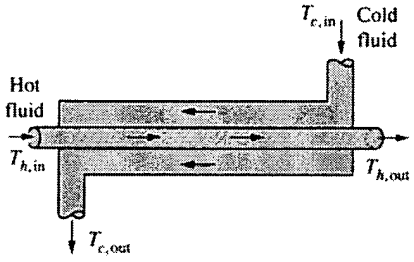


FIGURE 10-16

The variation of the fluid temperatures in a counter-flow double-pipe heat exchanger.

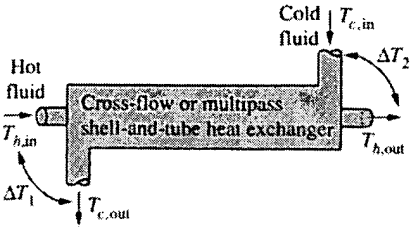


FIGURE 10-17

The determination of the heat transfer rate for cross-flow and multipass shell-and-tube heat exchangers using the correction factor.

Heat transfer rate:

$$\dot{Q} = UAF\Delta T_{lm,CF}$$

where $$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out}$$

$$\Delta T_2 = T_{h,out} - T_{c,in}$$

and $F = \ldots$ (Fig. 10–18)

A *condenser* or a *boiler* can be considered to be either a parallel- or counter-flow heat exchanger since both approaches give the same result.

Multipass and Cross-Flow Heat Exchangers: Use of a Correction Factor

The log mean temperature difference ΔT_{lm} relation developed earlier is limited to parallel-flow and counter-flow heat exchangers only. Similar relations are also developed for *cross-flow* and *multipass shell-and-tube* heat exchangers, but the resulting expressions are too complicated because of the complex flow conditions.

In such cases, it is convenient to relate the equivalent temperature difference to the log mean temperature difference relation for the counter-flow case as

$$\Delta T_{lm} = F\Delta T_{lm,CF} \tag{10-27}$$

where F is the **correction factor**, which depends on the *geometry* of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams. The $\Delta T_{lm,CF}$ is the log mean temperature difference for the case of a *counter-flow* heat exchanger with the same inlet and outlet temperatures and is determined from Equation 10-25 by taking $\Delta T_1 = T_{h,in} - T_{c,out}$ and $\Delta T_2 = T_{h,out} - T_{c,in}$ (Fig. 10-17).

The correction factor is less than unity for a cross-flow and multipass shell-and-tube heat exchanger. That is, $F \leq 1$. The limiting value of $F = 1$ corresponds to the counter-flow heat exchanger. Thus, the correction factor F for a heat exchanger is *a measure of deviation of the* ΔT_{lm} *from the corresponding values for the counter-flow case.*

The correction factor F for common cross-flow and shell-and-tube heat exchanger configurations is given in Figure 10-18 versus two temperature ratios P and R defined as

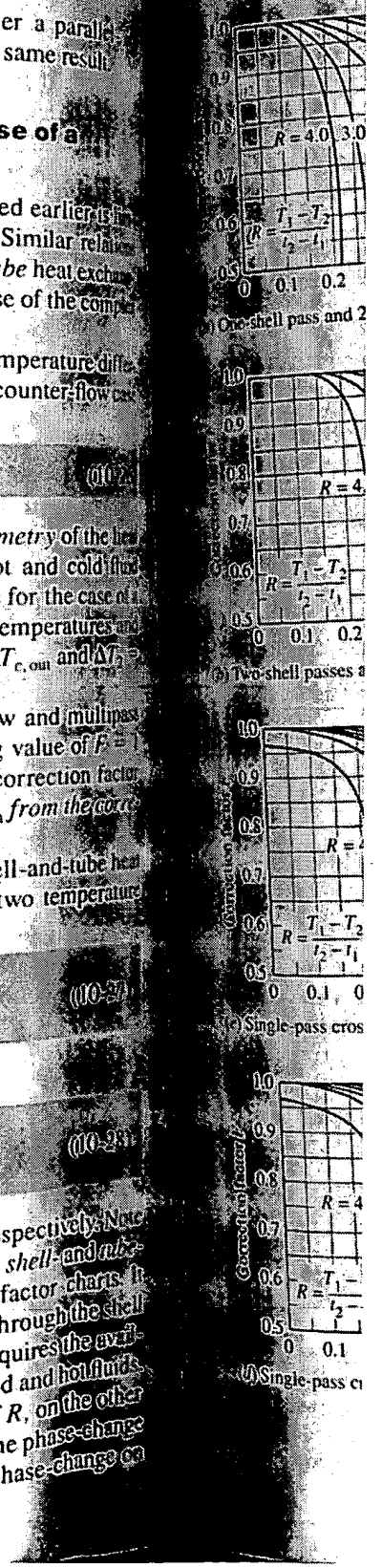
$$P = \frac{t_2 - t_1}{T_1 - t_1} \tag{10-28}$$

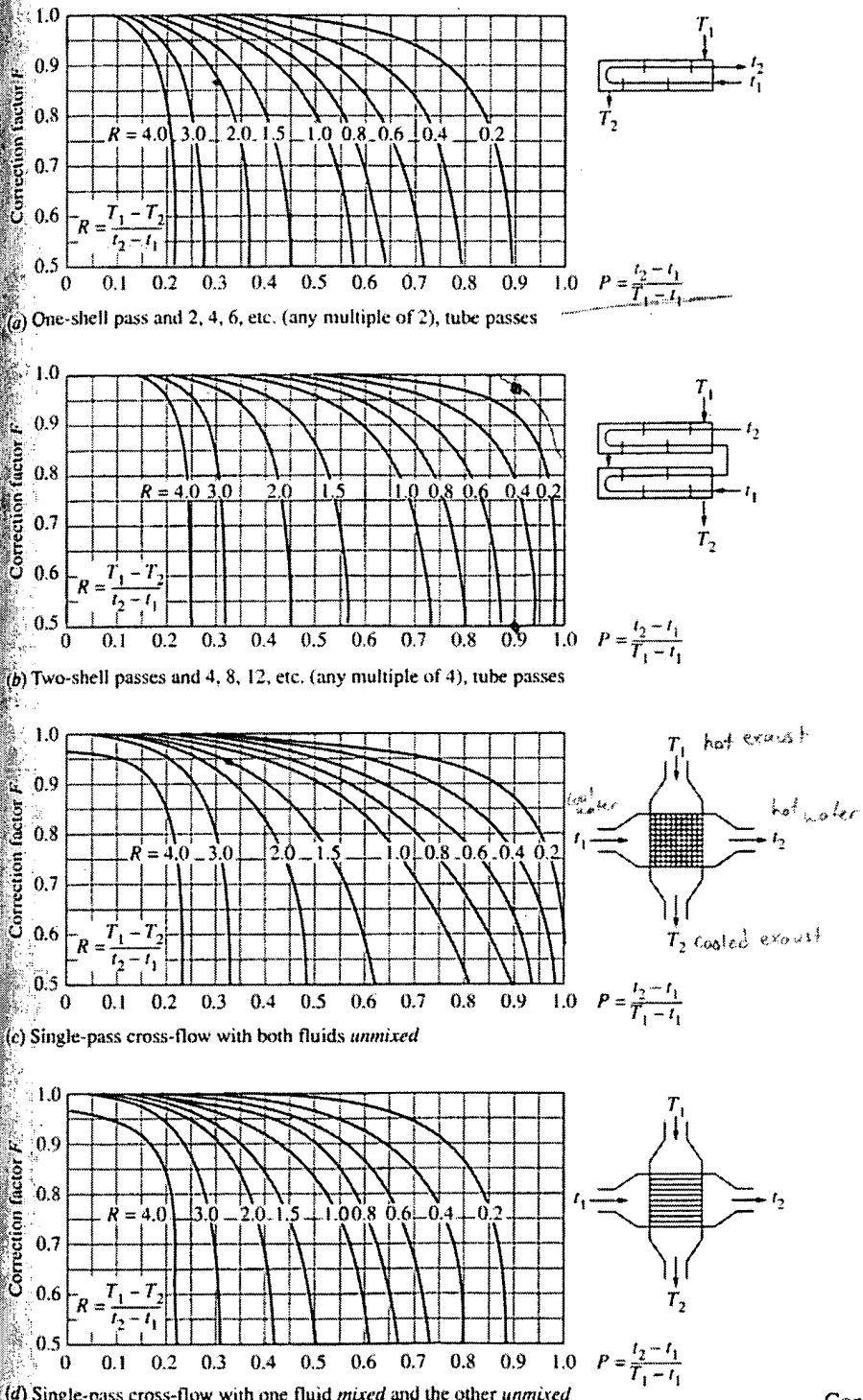
and

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}C_p)_{\text{tube side}}}{(\dot{m}C_p)_{\text{shell side}}} \tag{10-29}$$

where the subscripts 1 and 2 represent the *inlet* and *outlet*, respectively. Note that for a shell-and-tube heat exchanger, T and t represent the *shell-side* and *tube-side* temperatures, respectively, as shown in the correction factor charts. It makes no difference whether the hot or the cold fluid flows through the shell or the tube. The determination of the correction factor F requires the availability of the *inlet* and the *outlet* temperatures for both the cold and hot fluids.

Note that the value of P ranges from 0 to 1. The value of R, on the other hand, ranges from 0 to infinity, with $R = 0$ corresponding to the phase-change (condensation or boiling) on the shell-side and $R \to \infty$ to phase-change on





(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

(*c*) Single-pass cross-flow with both fluids *unmixed*

(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

FIGURE 10-18

Correction factor F charts for common shell-and-tube and cross-flow heat exchangers (from Bowman, Mueller, and Nagle, Ref. 2).

the tube side. The correction factor is $F = 1$ for both of these limiting cases. Therefore, the correction factor for a *condenser* or *boiler* is $F = 1$, regardless of the configuration of the heat exchanger.

EXAMPLE 10-3 The Condensation of Steam in a Condenser

Steam in the condenser of a steam power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C. The surface area of the tubes is 45 m² and the overall heat transfer coefficient is 2100 W/m² · °C. Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.

Solution Steam is condensed by cooling water in the condenser of a power plant. The mass flow rate of the cooling water and the rate of condensation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchager is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The heat of vaporization of water at 30°C is $h_{fg} = 2431$ kJ/kg and the specific heat of cold water at the average temperature of 18°C is $C_p = 4184$ J/kg.°C (Table A-9).

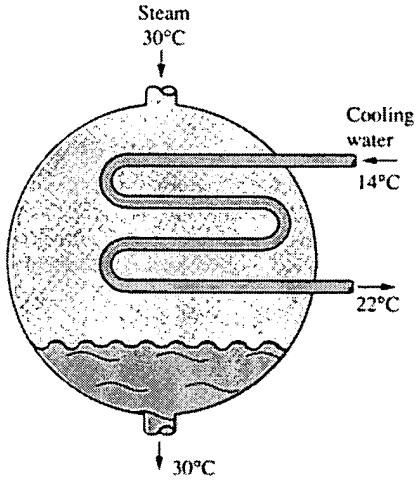


FIGURE 10-19
Schematic for Example 10-3.

Analysis The schematic of the condenser is given in Figure 10-19. The condenser can be treated as a counter-flow heat exchanger since the temperature of one of the fluids (the steam) remains constant.

The temperature difference between the steam and the cooling water at the two ends of the condenser is

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = (30 - 22)°\text{C} = 8°\text{C}$$
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = (30 - 14)°\text{C} = 16°\text{C}$$

That is, the temperature difference between the two fluids varies from 8°C at one end to 16°C at the other. The proper average temperature difference between the two fluids is the *logarithmic mean temperature difference* (not the arithmetic mean), which is determined from

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8 - 16}{\ln(8/16)} = 11.5°\text{C}$$

This is a little less than the arithmetic mean temperature difference, $\frac{1}{2}(8 + 16) = 12°\text{C}$. Then the heat transfer rate in the condenser is determined from

$$\dot{Q} = UA\Delta T_{\text{lm}} = (2100\ \text{W/m}^2 \cdot °\text{C})(45\ \text{m}^2)(11.5°\text{C}) = \mathbf{1{,}086{,}750\ W}$$

Therefore, the steam will lose heat at a rate of 1,086.75 kW as it flows through the condenser, and the cooling water will gain practically all of it, since the condenser is well insulated.

The mass flow rate of the cooling water and the rate of the condensation of the steam are determined from $\dot{Q} = [\dot{m}C(T_{\text{out}} - T_{\text{in}})]_{\text{cooling water}} = (\dot{m}h_{fg})_{\text{steam}}$ to be

$$\dot{m}_{\text{cooling water}} = \frac{\dot{Q}}{C_p(T_{\text{out}} - T_{\text{in}})} = \frac{1{,}086.75 \text{ kJ/s}}{(4.184 \text{ kJ/kg} \cdot °\text{C})(22 - 14)°\text{C}} = \mathbf{32.5 \text{ kg/s}}$$

$$\dot{m}_{\text{steam}} = \frac{\dot{Q}}{h_{fg}} = \frac{1{,}086.75 \text{ kJ/s}}{2431 \text{ kJ/kg}} = \mathbf{0.45 \text{ kg/s}}$$

Therefore, we need to circulate about 72 kg of cooling water for each 1 kg of steam condensing to remove the heat released during the condensation process.

EXAMPLE 10-4 Heating Water in a Counter-Flow Heat Exchanger

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m² · °C, determine the length of the heat exchanger required to achieve the desired heating.

Solution Water is heated in a counter-flow double-pipe heat exchanger by geothermal water. The required length of the heat exchanger is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties We take the specific heats of water and geothermal fluid to be 4.18 and 4.31 kJ/kg.°C, respectively.

Analysis The schematic of the heat exchanger is given in Figure 10-20. The rate of heat transfer in the heat exchanger can be determined from

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °\text{C})(80 - 20)°\text{C} = 301.0 \text{ kW}$$

Noting that all of this heat is supplied by the geothermal water, the outlet temperature of the geothermal water is determined to be

$$\dot{Q} = [\dot{m}C_p(T_{\text{in}} - T_{\text{out}})]_{\text{geothermal}} \longrightarrow T_{\text{out}} = T_{\text{in}} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 160°\text{C} - \frac{301.0 \text{ kW}}{(2 \text{ kg/s})(4.31 \text{ kJ/kg} \cdot °\text{C})}$$

$$= 125.1°\text{C}$$

Knowing the inlet and outlet temperatures of both fluids, the logarithmic mean temperature difference for this counter-flow heat exchanger becomes

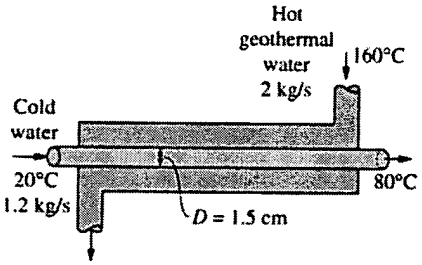


FIGURE 10-20
Schematic for Example 10-4.

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = (160 - 80)°\text{C} = 80°\text{C}$$
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = (125.1 - 20)°\text{C} = 105.1°\text{C}$$

and

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{80 - 105.1}{\ln(80/105.1)} = 92.0°\text{C}$$

Then the surface area of the heat exchanger is determined to be

$$\dot{Q} = UA\,\Delta T_{\text{lm}} \longrightarrow A = \frac{\dot{Q}}{U\Delta T_{\text{lm}}} = \frac{301{,}000\ \text{W}}{(640\ \text{W/m}^2 \cdot °\text{C})(92.0°\text{C})} = 5.11\ \text{m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A = \pi DL \longrightarrow L = \frac{A}{\pi D} = \frac{5.11\ \text{m}^2}{\pi(0.015\ \text{m})} = \mathbf{108.4\ m}$$

Discussion The inner tube of this counter-flow heat exchanger (and thus the heat exchanger itself) needs to be over 100 m long to achieve the desired heat transfer, which is impractical. In cases like this, we need to use a plate heat exchanger or a multipass shell-and-tube heat exchanger with multiple passes of tube bundles.

EXAMPLE 10-5 Heating of Glycerin in a Multipass Heat Exchanger

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (Fig. 10-21). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m^2 · °C on the glycerin (shell) side and 160 W/m^2 · °C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling occurs and (*b*) after fouling with a fouling factor of 0.0006 m^2 · °C/W occurs on the outer surfaces of the tubes.

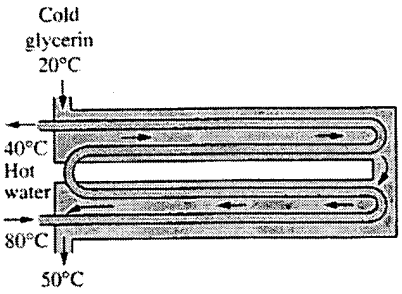


FIGURE 10-21
Schematic for Example 10-5.

Solution Glycerin is heated in a 2-shell passes and 4-tube passes heat exchanger by hot water. The rate of heat transfer for the cases of fouling and no fouling are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Analysis The tubes are said to be thin-walled, and thus it is reasonable to assume the inner surface area of the tubes to be equal to the outer surface area. Then the heat transfer surface area of this heat exchanger becomes

$$A = \pi DL = \pi(0.02\ \text{m})(60\ \text{m}) = 3.77\ \text{m}^2$$

The rate of heat transfer in this heat exchanger can be determined from

$$\dot{Q} = UAF\,\Delta T_{lm,\,CF}$$

where F is the correction factor and $\Delta T_{lm,\,CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are determined from

$$\Delta T_1 = T_{h,\,in} - T_{c,\,out} = (80 - 50)^\circ\text{C} = 30^\circ\text{C}$$

$$\Delta T_2 = T_{h,\,out} - T_{c,\,in} = (40 - 20)^\circ\text{C} = 20^\circ\text{C}$$

$$\Delta T_{lm,\,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{30 - 20}{\ln(30/20)} = 24.7^\circ\text{C}$$

and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75 \end{aligned}\right\} F = 0.91 \qquad \text{(Fig. 10-18}b\text{)}$$

(*a*) In the case of no fouling, the overall heat transfer coefficient U is determined from

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2\cdot^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2\cdot^\circ\text{C}}} = 21.6\ \text{W/m}^2\cdot^\circ\text{C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UAF\,\Delta T_{lm,\,CF} = (21.6\ \text{W/m}^2\cdot^\circ\text{C})(3.77\text{m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1730\ W}$$

(*b*) When there is fouling on one of the surfaces, the overall heat transfer coefficient U is determined from

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o} + R_f} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2\cdot^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2\cdot^\circ\text{C}} + 0.0006\ \text{m}^2\cdot^\circ\text{C/W}}$$

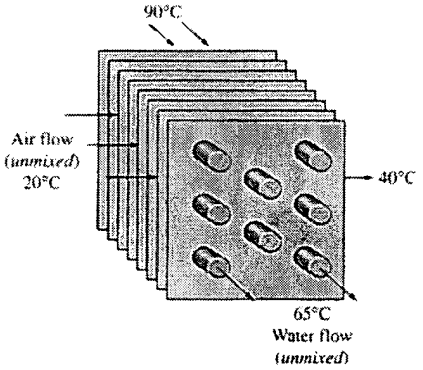
$$= 21.3\ \text{W/m}^2\cdot^\circ\text{C}$$

The rate of heat transfer in this case becomes

$$\dot{Q} = UAF\Delta T_{lm,\,CF} = (21.3\ \text{W/m}^2\cdot^\circ\text{C})(3.77\text{m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1805\ W}$$

Discussion Note that the rate of heat transfer decreases as a result of fouling, as expected. The decrease is not dramatic, however, because of the relatively low convection heat transfer coefficients involved.

FIGURE 10-22
Schematic for Example 10-6.



EXAMPLE 10-6 Cooling of an Automotive Radiator

A test is conducted to determine the overall heat transfer coefficient in an automotive radiator that is a compact cross-flow water-to-air heat exchanger with both fluids (air and water) unmixed (Fig. 10-22). The radiator has 40 tubes of internal diameter 0.5 cm and length 65 cm in a closely spaced plate-finned matrix. Hot water enters the tubes at 90°C at a rate of 0.6 kg/s and leaves at 65°C. Air flows across the radiator through the interfin spaces and is heated from 20°C to 40°C. Determine the overall heat transfer coefficient U_i of this radiator based on the inner surface area of the tubes.

Solution During an experiment involving an automotive radiator, the inlet and exit temperatures of water and air and the mass flow rate of water are measured. The overall heat transfer coefficient based on the inner surface area is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Changes in the kinetic and potential energies of fluid streams are negligible. **3** Fluid properties are constant.

Properties The specific heat of water at the average temperature of (90 + 65)/2 = 77.5°C is 4.195 kJ/kg · °C.

Analysis The rate of heat transfer in this radiator from the hot water to the air is determined from an energy balance on water flow,

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{water} = (0.6 \text{ kg/s})(4.195 \text{ kJ/kg} \cdot °\text{C})(90 - 65)°\text{C} = 62.93 \text{ kW}$$

The tube-side heat transfer area is the total surface area of the tubes and is determined from

$$A_i = n\pi D_i L = (40)\pi(0.005 \text{ m})(0.65 \text{ m}) = 0.408 \text{ m}^2$$

Knowing the rate of heat transfer and the surface area, the overall heat transfer coefficient in this heat exchanger can be determined from

$$\dot{Q} = U_i A_i F \Delta T_{lm,CF} \longrightarrow U_i = \frac{\dot{Q}}{A_i F \Delta T_{lm,CF}}$$

where F is the correction factor and $\Delta T_{lm,CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are found to be

$$\Delta T_1 = T_{h,in} - T_{c,out} = (90 - 40)°\text{C} = 50°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = (65 - 20)°\text{C} = 45°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{50 - 45}{\ln(50/45)} = 47.6°\text{C}$$

and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{65 - 90}{20 - 90} = 0.36 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 40}{65 - 90} = 0.80 \end{aligned}\right\} F = 0.97 \quad \text{(Fig. 10-18c)}$$

Substituting, the overall heat transfer coefficient U_i is determined to be

$$U_i = \frac{\dot{Q}}{A_i F \Delta T_{lm,CF}} = \frac{62{,}930 \text{ W}}{(0.408 \text{ m}^2)(0.97)(47.6°\text{C})} = 3341 \text{ W/m}^2 \cdot °\text{C}$$

Note that the overall heat transfer coefficient on the air side would be much lower because of the large surface area involved on that side.

10-5 ■ THE EFFECTIVENESS–NTU METHOD

The log mean temperature difference (LMTD) method discussed in the previous section is easy to use in heat exchanger analysis when the inlet and the outlet temperatures of the hot and cold fluids are known or can be determined

from an energy balance. Once ΔT_{lm}, the mass flow rates, and the overall heat transfer coefficient are available, the heat transfer surface area of the heat exchanger can be determined from

$$\dot{Q} = UA\,\Delta T_{lm}$$

Therefore, the LMTD method is very suitable for determining the *size* of a heat exchanger to realize prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified.

With the LMTD method, the task is to *select* a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to be followed by the selection process is as follows:

1. Select the type of heat exchanger suitable for the application.
2. Determine any unknown inlet or outlet temperature and the heat transfer rate using an energy balance.
3. Calculate the log mean temperature difference ΔT_{lm} and the correction factor F, if necessary.
4. Obtain (select or calculate) the value of the overall heat transfer coefficient U.
5. Calculate the heat transfer surface area A.

The task is completed by selecting a heat exchanger that has a heat transfer surface area equal to or larger than A.

A second kind of problem encountered in heat exchanger analysis is the determination of the *heat transfer rate* and the *outlet temperatures* of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the *type* and *size* of the heat exchanger are specified. The heat transfer surface area A of the heat exchanger in this case is known, but the *outlet temperatures* are not. Here the task is to determine the heat transfer performance of a specified heat exchanger or to determine if a heat exchanger available in storage will do the job.

The LMTD method could still be used for this alternative problem, but the procedure would require tedious iterations, and thus it is not practical. In an attempt to eliminate the iterations from the solution of such problems, Kays and London came up with a new method in 1955 called the **effectiveness-NTU method,** which greatly simplified heat exchanger analysis.

This new method is based on a dimensionless parameter called the **heat transfer effectiveness** ε, defined as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}} \tag{10-29}$$

The *actual* heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as

$$\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) = C_h(T_{h,\text{in}} - T_{h,\text{out}}) \tag{10-30}$$

where $C_c = \dot{m}_c C_{pc}$ and $C_h = \dot{m}_c C_{ph}$ are the heat capacity rates of the cold and the hot fluids, respectively.

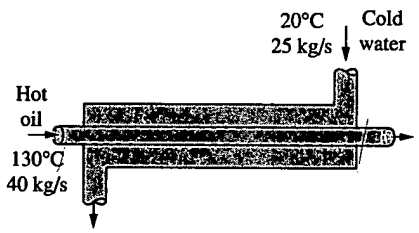
To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the *maximum temperature difference* in a heat exchanger is the difference between the *inlet* temperatures of the hot and cold fluids. That is,

$$\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} \tag{10-31}$$

The heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid. These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical (i.e., $C_c = C_h$). When $C_c \neq C_h$, which is usually the case, the fluid with the *smaller* heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt. Therefore, the maximum possible heat transfer rate in a heat exchanger is (Fig. 10-23)

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) \tag{10-32}$$

where $C_{\min}$ is the smaller of $C_h = \dot{m}_h C_{ph}$ and $C_c = \dot{m}_c C_{pc}$. This is further clarified by the following example.



$C_c = \dot{m}_c C_{pc} = 104.5$ kW/°C

$C_h = \dot{m}_c C_{ph} = 92$ kW/°C

$C_{\min} = 92$ kW/°C

$\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} = 110$°C

$\dot{Q}_{\max} = C_{\min}\,\Delta T_{\max} = 10{,}120$ kW

FIGURE 10-23
The determination of the maximum rate of heat transfer in a heat exchanger.

EXAMPLE 10-7 Upper Limit for Heat Transfer in a Heat Exchanger

Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at $C_p = 4.18$ kJ/kg · °C, determine the maximum heat transfer rate and the outlet temperatures of the cold and the hot water streams for this limiting case.

Solution Cold and hot water streams enter a heat exchanger at specified temperatures and flow rates. The maximum rate of heat transfer in the heat exchanger is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heat of water is given to be $C_p = 4.18$ kJ/kg · °C.

Analysis A schematic of the heat exchanger is given in Figure 10-24. The heat capacity rates of the hot and cold fluids are determined from

$$C_h = \dot{m}_h C_{ph} = (2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °\text{C}) = 8.36 \text{ kW/°C}$$

and

$$C_c = \dot{m}_c C_{pc} = (8 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °\text{C}) = 33.44 \text{ kW/°C}$$

Therefore

$$C_{\min} = C_h = 8.36 \text{ kW/°C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate is determined from Equation 10-32 to be

$$\begin{aligned} \dot{Q}_{\max} &= C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) \\ &= (8.36 \text{ kW/°C})(70 - 10)°\text{C} \\ &= \mathbf{501.6 \text{ kW}} \end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 501.6 kW. This value would be approached in a counter-flow heat exchanger with a *very large* heat transfer surface area.

The maximum temperature difference in this heat exchanger is $\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} = (70 - 10)°\text{C} = 60°\text{C}$. Therefore, the hot water cannot be cooled by more than 60°C (to 10°C) in this heat exchanger, and the cold water cannot be heated by more than 60°C (to 70°C), no matter what we do. The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

$$\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) \longrightarrow T_{c,\text{out}} = T_{c,\text{in}} + \frac{\dot{Q}}{C_c} = 10°\text{C} + \frac{501.6 \text{ kW}}{33.44 \text{ kW/°C}} = \mathbf{25°C}$$

$$\dot{Q} = C_h(T_{h,\text{in}} - T_{h,\text{out}}) \longrightarrow T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{C_h} = 70°\text{C} - \frac{501.6 \text{ kW}}{8.38 \text{ kW/°C}} = \mathbf{10°C}$$

Discussion Note that the hot water is cooled to the limit of 10°C (the inlet temperature of the cold water stream), but the cold water is heated to 25°C only when maximum heat transfer occurs in the heat exchanger. This is not surprising, since the mass flow rate of the hot water is only one-fourth that of the cold water, and, as a result, the temperature of the cold water increases by 0.25°C for each 1°C drop in the temperature of the hot water.

You may be tempted to think that the cold water should be heated to 70°C in the limiting case of maximum heat transfer. But this will require the temperature of the hot water to drop to −170°C (below 10°C), which is impossible. Therefore, heat transfer in a heat exchanger reaches its maximum value when the fluid with the smaller heat capacity rate (or the smaller mass flow rate when both fluids have the same specific heat value) experiences the maximum temperature change. This example explains why we use $C_{\min}$ in the evaluation of $\dot{Q}_{\max}$ instead of $C_{\max}$.

We can show that the hot water will leave at the inlet temperature of the cold water and vice versa in the limiting case of maximum heat transfer when the mass flow rates of the hot and cold water streams are identical (Fig. 10-25). We can also show that the outlet temperature of the cold water will reach the 70°C limit when the mass flow rate of the hot water is greater than that of the cold water.

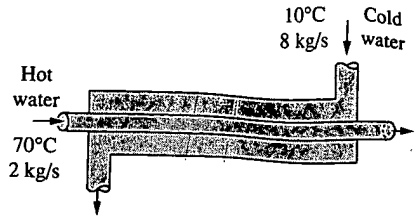
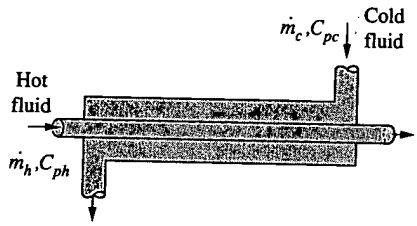


FIGURE 10-24
Schematic for Example 10-7.



$$\dot{Q} = \dot{m}_h C_{ph} \Delta T_h = \dot{m}_c C_{pc} \Delta T_c$$

If $\dot{m}_c C_{pc} = \dot{m}_h C_{ph}$

then $\Delta T_h = \Delta T_c$

FIGURE 10-25
The temperature rise of the cold fluid in a heat exchanger will be equal to the temperature drop of the hot fluid when the mass flow rates and the specific heats of the hot and cold fluids are identical.

The determination of $\dot{Q}_{max}$ requires the availability of the *inlet temperature* of the hot and cold fluids and their *mass flow rates*, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate $\dot{Q}$ can be determined from

$$\dot{Q} = \varepsilon \dot{Q}_{max} = \varepsilon C_{min}(T_{h,in} - T_{c,in}) \quad (10\text{-}33)$$

Therefore, the effectiveness of a heat exchanger enables us to determine the heat transfer rate without knowing the *outlet temperatures* of the fluids.

The effectiveness of a heat exchanger depends on the *geometry* of the heat exchanger as well as the *flow arrangement*. Therefore, different types of heat exchangers have different effectiveness relations. Below we illustrate the development of the effectiveness ε relation for the double-pipe *parallel-flow* heat exchanger.

Equation 10-23 developed in the previous section for a parallel-flow heat exchanger can be rearranged as

$$\ln \frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}} = -\frac{UA}{C_c}\left(1 + \frac{C_c}{C_h}\right) \quad (10\text{-}34)$$

Also, solving Equation 10-30 for $T_{h,out}$ gives

$$T_{h,out} = T_{h,in} - \frac{C_c}{C_h}(T_{c,out} - T_{c,in}) \quad (10\text{-}35)$$

Substituting this relation into Equation 10-34 after adding and subtracting $T_{c,in}$ gives

$$\ln \frac{T_{h,in} - T_{c,in} + T_{c,in} - T_{c,out} - \frac{C_c}{C_h}(T_{c,out} - T_{c,in})}{T_{h,in} - T_{c,in}} = -\frac{UA}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

which simplifies to

$$\ln\left[1 - \left(1 + \frac{C_c}{C_h}\right)\frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}}\right] = -\frac{UA}{C_c}\left(1 + \frac{C_c}{C_h}\right) \quad (10\text{-}36)$$

We now manipulate the definition of effectiveness to obtain

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{min}(T_{h,in} - T_{c,in})} \longrightarrow \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}} = \varepsilon\frac{C_{min}}{C_c}$$

Substituting this result into Equation 10-36 and solving for ε gives the following relation for the effectiveness of a *parallel-flow* heat exchanger:

$$\varepsilon_{parallel\ flow} = \frac{1 - \exp\left[-\frac{UA}{C_c}\left(1 + \frac{C_c}{C_h}\right)\right]}{\left(1 + \frac{C_c}{C_h}\right)\frac{C_{min}}{C_c}} \quad (10\text{-}37)$$

Taking either C_c or C_h to be C_{min} (both approaches give the same result), the relation above can be expressed more conveniently as

$$\varepsilon_{\text{parallel flow}} = \frac{1 - \exp\left[-\frac{UA}{C_{min}}\left(1 + \frac{C_{min}}{C_{max}}\right)\right]}{1 + \frac{C_{min}}{C_{max}}} \tag{10-38}$$

Again C_{min} is the *smaller* heat capacity ratio and C_{max} is the larger one, and it makes no difference whether C_{min} belongs to the hot or cold fluid.

Effectiveness relations of the heat exchangers typically involve the *dimensionless* group UA/C_{min}. This quantity is called the **number of transfer units NTU** and is expressed as

$$\text{NTU} = \frac{UA}{C_{min}} = \frac{UA}{(\dot{m}c_p)_{min}} \tag{10-39}$$

where U is the overall heat transfer coefficient and A is the heat transfer surface area of the heat exchanger. Note that NTU is proportional to A. Therefore, for specified values of U and C_{min}, the value of NTU *is a measure of the heat transfer surface area A*. Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called **capacity ratio** C as

$$C = \frac{C_{min}}{C_{max}} \tag{10-40}$$

It can be shown that the effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio C. That is,

$$\varepsilon = \text{function}\,(UA/C_{min},\ C_{min}/C_{max}) = \text{function}\,(\text{NTU}, C)$$

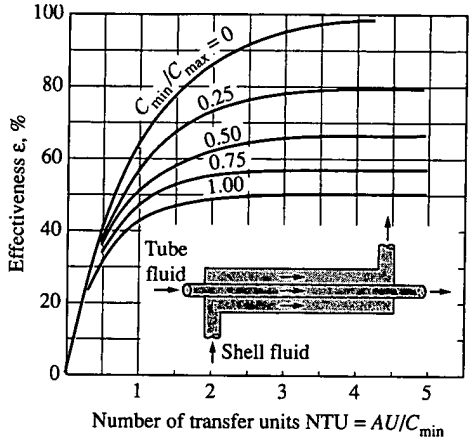
Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in Table 10-4. The effectivenesses of some common types of heat exchangers are also plotted in Figure 10-26. More extensive effectiveness charts and relations are available in the literature. The dashed lines in Figure 10-26*f* are for the case of C_{min} unmixed and C_{max} mixed, and the solid lines are for the opposite case. The analytic relations for the effectiveness give more accurate results than the charts, since reading errors in charts are unavoidable, and the relations are very suitable for computerized design analysis of heat exchangers.

We make the following observations from the effectiveness relations and charts given above:

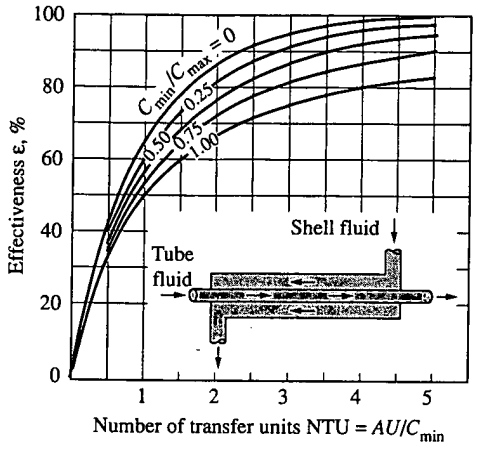
1 The value of the effectiveness ranges from 0 to 1. It increases rapidly with NTU for small values (up to about NTU = 1.5) but rather slowly for larger values. Therefore, the use of a heat exchanger with a large NTU

FIGURE 10-26

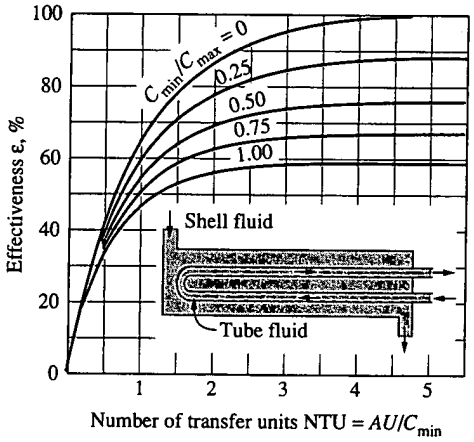
Effectiveness for heat exchangers (from Kays and London, Ref. 7).



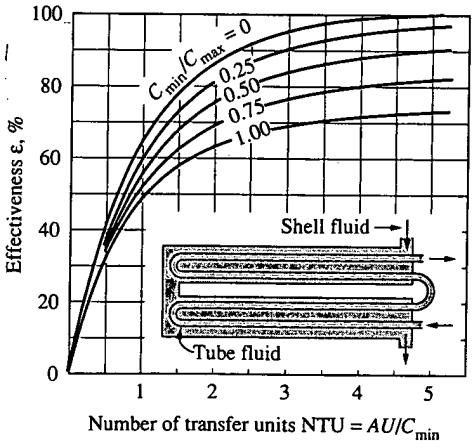
(*a*) Parallel-flow



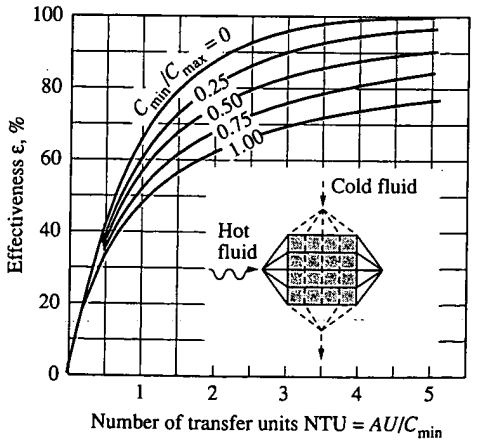
(*b*) Counter-flow



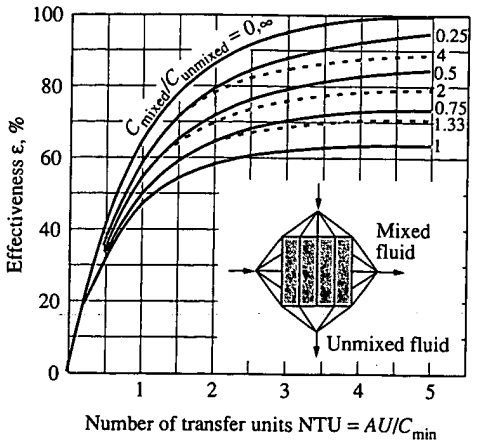
(*c*) One-shell pass and 2, 4, 6, tube passes



(*d*) Two-shell passes and 4, 8, 12, tube passes



(*e*) Cross-flow with both fluids unmixed



(*f*) Cross-flow with one fluid mixed and the other unmixed

TABLE 10-4

Effectiveness relatio
and $C = C_{min}/C_{max}$ =

Heat exchanger type
1 Double pipe:
Parallel-flow
Counter-flow
2 Shell and tube:
One-shell pass
2, 4, ... tube passes
3 Cross-flow (single-pass)
Both fluids unmixed
C_{max} mixed, C_{min} unmixed
C_{min} mixed, C_{max} unmixed
4 All heat exchangers with $C = 0$

Source: Kays and Lond

(usually larger tha
since a large increa
effectiveness. Thu
highly desirable fr
from an economic

2 For a given N
heat exchanger ha
flow heat exchang
lowest effectivene
(Fig. 10-27).

3 The effectiven
C for NTU values

4 The value of
NTU, the effectiv
$C = 1$. The case

TABLE 10-4

Effectiveness relations for heat exchangers: NTU = UA/C_{min} and $C = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$

Heat exchanger type	Effectiveness relation
1 Double pipe:	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + C)]}{1 + C}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - C)]}{1 - C\exp[-\text{NTU}(1 - C)]}$
2 Shell-and-tube:	
One-shell pass 2, 4, ... tube passes	$\varepsilon = 2\left\{1 + C + \sqrt{1 + C^2}\,\dfrac{1 + \exp[-\text{NTU}\sqrt{1 + C^2}]}{1 - \exp[-\text{NTU}\sqrt{1 + C^2}]}\right\}^{-1}$
3 Cross-flow (single-pass)	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{C}[\exp(-C\,\text{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{C}(1 - \exp\{1 - C[1 - \exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{C}[1 - \exp(-C\,\text{NTU})]\right\}$
4 All heat exchangers with $C = 0$	$\varepsilon = 1 - \exp(-\text{NTU})$

Source: Kays and London, Ref. 7.

(usually larger than 3) and thus a large size cannot be justified economically, since a large increase in NTU in this case corresponds to a small increase in effectiveness. Thus, a heat exchanger with a very high effectiveness may be highly desirable from a heat transfer point of view but rather undesirable from an economical point of view

2 For a given NTU and capacity ratio $C = C_{min}/C_{max}$, the *counter-flow* heat exchanger has the *highest* effectiveness, followed closely by the cross-flow heat exchangers with both fluids unmixed. As you might expect, the lowest effectiveness values are encountered in parallel-flow heat exchangers (Fig. 10-27).

3 The effectiveness of a heat exchanger is independent of the capacity ratio C for NTU values of less than about 0.3.

4 The value of the capacity ratio C ranges between 0 and 1. For a given NTU, the effectiveness becomes a *maximum* for $C = 0$ and a *minimum* for $C = 1$. The case $C = C_{min}/C_{max} \to 0$ corresponds to $C_{max} \to \infty$, which is

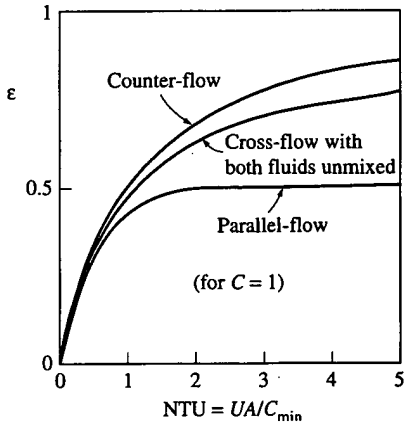


FIGURE 10-27

For a specified NTU and capacity ratio C, the counter-flow heat exchanger has the highest effectiveness and the parallel-flow the lowest.

CHAPTER 10
Heat Exchangers

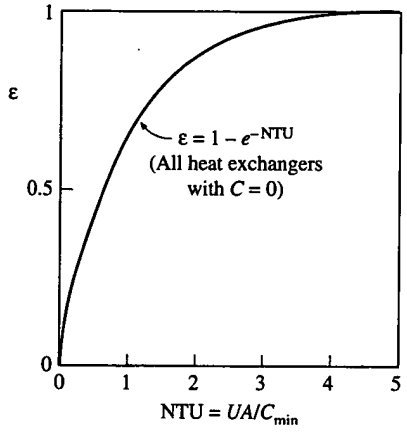


FIGURE 10-28

The effectiveness relation reduces to $\varepsilon = \varepsilon_{max} = 1 - \exp(-\text{NTU})$ for all heat exchangers when the capacity ratio $C = 0$.

realized during a phase-change process in a *condenser* or *boiler*. All effectiveness relations in this case reduce to

$$\varepsilon = \varepsilon_{max} = 1 - \exp(-\text{NTU}) \tag{10-41}$$

regardless of the type of heat exchanger (Fig. 10-28). Note that the temperature of the condensing or boiling fluid remains constant in this case. The effectiveness is the *lowest* in the other limiting case of $C = C_{min}/C_{max} = 1$, which is realized when the heat capacity rates of the two fluids are equal.

Once the quantities $C = C_{min}/C_{max}$ and $\text{NTU} = UA/C_{min}$ have been evaluated, the effectiveness ε can be determined from either the charts or (preferably) the effectiveness relation for the specified type of heat exchanger. Then the rate of heat transfer $\dot{Q}$ and the outlet temperatures $T_{h,out}$ and $T_{c,out}$ can be determined from Equations 10-33 and 10-30, respectively. Note that the analysis of heat exchangers with unknown outlet temperatures is a straightforward matter with the effectiveness-NTU method but requires rather tedious iterations with the LMTD method.

We mentioned earlier that when all the inlet and outlet temperatures are specified, the *size* of the heat exchanger can easily be determined using the LMTD method. Alternatively, it can also be determined from the effectiveness-NTU method by first evaluating the effectiveness ε from its definition (Eq. 10-29) and then the NTU from the appropriate NTU relation in Table 10-5.

Note that the relations in Table 10-5 are equivalent to those in Table 10-4. Both sets of relations are given for convenience. The relations in Table 10-4 give the effectiveness directly when NTU is known, and the relations in Table 10-5 give the NTU directly when the effectiveness ε is known.

EXAMPLE 10-8 Using the Effectiveness-NTU Method

Repeat Example 10-4, which was solved with the LMTD method, using the effectiveness-NTU method.

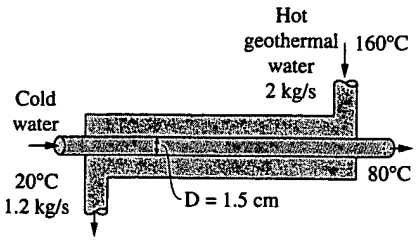


FIGURE 10-29

Schematic for Example 10-8.

Solution The schematic of the heat exchanger is redrawn in Figure 10-29, and the same assumptions are utilized.

Analysis In the effectiveness-NTU method, we first determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (2\ \text{kg/s})(4.31\ \text{kJ/kg}\cdot°\text{C}) = 8.62\ \text{kW/°C}$$
$$C_c = \dot{m}_c C_{pc} = (1.2\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot°\text{C}) = 5.02\ \text{kW/°C}$$

Therefore,

$$C_{min} = C_c = 5.02\ \text{kW/°C}$$

and

$$C = C_{min}/C_{max} = 5.02/8.62 = 0.583$$

Then the maximum heat transfer rate is determined from Equation 10-32 to be

$$\dot{Q}_{max} = C_{min}(T_{h,in} - T_{c,in})$$
$$= (5.02\ \text{kW/°C})(160 - 20)°\text{C}$$
$$= 702.8\ \text{kW}$$

TABLE 10-5
NTU relations f
and $C = C_{min}/C$
Heat exchan
1 Double-pipe:
Parallel-flow
Counter-flow
2 Shell and tub
One-shell pa
2, 4, . . . tube
3 Cross-flow (s
C_{max} mixed,
C_{min} unmixec
C_{min} mixed,
C_{max} unmixe
4 All heat exch
with $C = 0$
Source: Kays and

That is, the m
702.8 kW. The
$Q = [\dot{m}C_p(T_o$
Thus, the effec
Knowing the e
determined frc
choose the latt
$\text{NTU} = \frac{}{C}$
Then the heat
NTU =
To provide this

TABLE 10-5

NTU relations for heat exchangers $\text{NTU} = UA/C_{\min}$ **and** $C = C_{\min}/C_{\max} = (\dot{m}C_p)_{\min}/(\dot{m}C_p)_{\max}$

Heat exchanger type	NTU relation
1 Double-pipe:	
Parallel-flow	$\text{NTU} = -\dfrac{\ln[1-\varepsilon(1+C)]}{1+C}$
Counter-flow	$\text{NTU} = \dfrac{1}{C-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon C-1}\right)$
2 Shell and tube:	
One-shell pass 2, 4, . . . tube passes	$\text{NTU} = -\dfrac{1}{\sqrt{1+C^2}}\ln\left(\dfrac{2/\varepsilon-1-C-\sqrt{1+C^2}}{2/\varepsilon-1-C+\sqrt{1+C^2}}\right)$
3 Cross-flow (single-pass)	
$C_{\max}$ mixed, $C_{\min}$ unmixed	$\text{NTU} = -\ln\left[1+\dfrac{\ln(1-\varepsilon C)}{C}\right]$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\text{NTU} = -\dfrac{\ln[C\ln(1-\varepsilon)+1]}{C}$
4 All heat exchangers with $C = 0$	$\text{NTU} = -\ln(1-\varepsilon)$

Source: Kays and London, Ref. 7.

That is, the maximum possible heat transfer rate in this heat exchanger is 702.8 kW. The actual rate of heat transfer in the heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot°\text{C})(80-20)°\text{C} = 301.0\ \text{kW}$$

Thus the effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{301.0\ \text{kW}}{702.8\ \text{kW}} = 0.428$$

Knowing the effectiveness, the NTU of this counter-flow heat exchanger can be determined from Figure 10-26*b* or the appropriate relation from Table 10-5. We choose the latter approach for greater accuracy:

$$\text{NTU} = \frac{1}{C-1}\ln\left(\frac{\varepsilon-1}{\varepsilon C-1}\right) = \frac{1}{0.583-1}\ln\left(\frac{0.428-1}{0.428\times 0.583-1}\right) = 0.651$$

Then the heat transfer surface area becomes

$$\text{NTU} = \frac{UA}{C_{\min}} \longrightarrow A = \frac{\text{NTU}\,C_{\min}}{U} = \frac{(0.651)(5020\ \text{W/°C})}{640\ \text{W/m}^2\cdot°\text{C}} = 5.11\ \text{m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A = \pi DL \longrightarrow L = \frac{A}{\pi D} = \frac{5.11\ \text{m}^2}{\pi(0.015\ \text{m})} = \mathbf{108.4\ m}$$

Discussion Note that we obtained the same result with the effectiveness-NTU method in a systematic and straightforward manner.

EXAMPLE 10-9 Cooling Hot Oil by Water in a Multipass Heat Exchanger

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m² · °C. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

Solution Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** The thickness of the tube is negligible since it is thin-walled. **4** Changes in the kinetic and potential energies of fluid streams are negligible. **5** The overall heat transfer coefficient is constant and uniform.

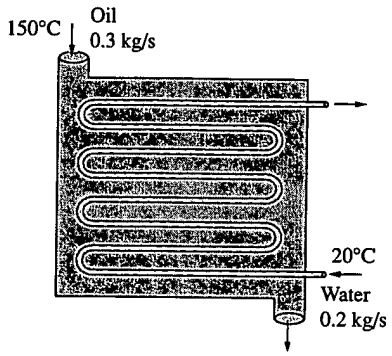


FIGURE 10-30
Schematic for Example 10-9.

Analysis The schematic of the heat exchanger is given in Figure 10-30. The outlet temperatures are not specified, and they cannot be determined from an energy balance. The use of the LMTD method in this case will involve tedious iterations, and thus the ε-NTU method is indicated. The first step in the ε-NTU method is to determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (0.3 \text{ kg/s})(2.13 \text{ kJ/kg} \cdot °\text{C}) = 0.639 \text{ kW/°C}$$
$$C_c = \dot{m}_c C_{pc} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °\text{C}) = 0.836 \text{ kW/°C}$$

Therefore,

$$C_{\min} = C_h = 0.639 \text{ kW/°C}$$

and

$$C = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

Then the maximum heat transfer rate is determined from Equation 10-32 to be

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})$$
$$= (0.639 \text{ kW/°C})(150 - 20)°\text{C} = 83.1 \text{ kW}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 83.1 kW. The heat transfer surface area is

$$A = n(\pi DL) = 8\pi(0.014 \text{ m})(5 \text{ m}) = 1.76 \text{ m}^2$$

Then the NTU of this heat exchanger becomes

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{(310\ \text{W/m}^2 \cdot °\text{C})(1.76\ \text{m}^2)}{639\ \text{W/°C}} = 0.853$$

The effectiveness of this heat exchanger corresponding to $C = 0.764$ and NTU = 0.853 is determined from Figure 10-26*c* to be

$$\varepsilon = 0.47$$

We could also determine the effectiveness from the third relation in Table 10-4 more accurately but with more labor. Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.47)(83.1\ \text{kW}) = \mathbf{39.1\ kW}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) \longrightarrow T_{c,\text{out}} = T_{c,\text{in}} + \frac{\dot{Q}}{C_c}$$

$$= 20°\text{C} + \frac{39.1\ \text{kW}}{0.836\ \text{kW/°C}} = \mathbf{66.8°C}$$

$$\dot{Q} = C_h(T_{h,\text{in}} - T_{h,\text{out}}) \longrightarrow T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{C_h}$$

$$= 150°\text{C} - \frac{39.1\ \text{kW}}{0.639\ \text{kW/°C}} = \mathbf{88.8°C}$$

Therefore, the temperature of the cooling water will rise from 20°C to 66.8°C as it cools the hot oil from 150°C to 88.8°C in this heat exchanger.

10-6 ■ THE SELECTION OF HEAT EXCHANGERS

Heat exchangers are complicated devices, and the results obtained with the simplified approaches presented above should be used with care. For example, we assumed that the overall heat transfer coefficient U is constant throughout the heat exchanger and that the convection heat transfer coefficients can be predicted using the convection correlations. However, it should be kept in mind that the uncertainty in the predicted value of U can even exceed 30 percent. Thus, it is natural to tend to overdesign the heat exchangers in order to avoid unpleasant surprises.

Heat transfer enhancement in heat exchangers is usually accompanied by *increased pressure drop,* and thus *higher pumping power.* Therefore, any gain from the enhancement in heat transfer should be weighed against the cost of the accompanying pressure drop. Also, some thought should be given to which fluid should pass through the tube side and which through the shell side. Usually, the more viscous fluid is more suitable for the shell side (larger passage area and thus lower pressure drop) and the fluid with the higher pressure for the tube side.

Engineers in industry often find themselves in a position to select heat exchangers to accomplish certain heat transfer tasks. Usually, the goal is to

heat or cool a certain fluid at a known mass flow rate and temperature to a desired temperature. Thus, the rate of heat transfer in the prospective heat exchanger is

$$\dot{Q}_{max} = \dot{m}C_p(T_{in} - T_{out})$$

which gives the heat transfer requirement of the heat exchanger before having any idea about the heat exchanger itself.

An engineer going through catalogs of heat exchanger manufacturers will be overwhelmed by the type and number of readily available off-the-shelf heat exchangers. The proper selection depends on several factors.

Heat Transfer Rate

This is the most important quantity in the selection of a heat exchanger. A heat exchanger should be capable of transferring heat at the specified rate in order to achieve the desired temperature change of the fluid at the specified mass flow rate.

Cost

Budgetary limitations usually play the most important role in the selection of heat exchangers, except for some specialized cases where "money is no object." An off-the-shelf heat exchanger has a definite cost advantage over those made to order. However, in some cases, none of the existing heat exchangers will do, and it may be necessary to undertake the expensive and time-consuming task of designing and manufacturing a heat exchanger from scratch to suit the needs. This is often the case when the heat exchanger is an integral part of the overall device to be manufactured.

The operation and maintenance costs of the heat exchanger are also important considerations in assessing the overall cost.

Pumping Power

In a heat exchanger, both fluids are usually forced to flow by pumps or fans that consume electrical power. The annual cost of electricity associated with the operation of the pumps and fans can be determined from

$$\text{Operating cost} = (\text{Pumping power, kW}) \times (\text{Hours of operation, h}) \times (\text{Price of electricity, \$/kWh})$$

where the pumping power is the total electrical power consumed by the motors of the pumps and fans. For example, a heat exchanger that involves a 1-hp pump and a $\frac{1}{3}$-hp fan (1 hp = 0.746 kW) operating 8 h a day and 5 days a week will consume 2017 kWh of electricity per year, which will cost $161.4 at an electricity cost of 8 cents/kWh.

Minimizing the pressure drop and the mass flow rate of the fluids will *minimize* the operating cost of the heat exchanger, but it will *maximize* the size of the heat exchanger and thus the initial cost. As a rule of thumb, doubling the mass flow rate will reduce the initial cost by *half* but will increase the pumping power requirements by a factor of roughly *eight*.

Typically, fluid velocities encountered in heat exchangers range between 0.7 and 7 m/s for liquids and between 3 and 30 m/s for gases. Low velocities are helpful in avoiding erosion, tube vibrations, and noise as well as pressure drop.

Size and Weight

Normally, the *smaller* and the *lighter* the heat exchanger, the better it is. This is especially the case in the *automotive* and *aerospace* industries, where size and weight requirements are most stringent. Also, a larger heat exchanger normally carries a higher price tag. The space available for the heat exchanger in some cases limits the length of the tubes that can be used.

Type

The type of heat exchanger to be selected depends primarily on the type of *fluids* involved, the *size* and *weight* limitations, and the presence of any *phase-change* processes. For example, a heat exchanger is suitable to cool a liquid by a gas if the surface area on the gas side is many times that on the liquid side. On the other hand, a plate or shell-and-tube heat exchanger is very suitable for cooling a liquid by another liquid.

Materials

The materials used in the construction of the heat exchanger may be an important consideration in the selection of heat exchangers. For example, the thermal and structural *stress effects* need not be considered at pressures below 15 atm or temperatures below 150°C. But these effects are major considerations above 70 atm or 550°C and seriously limit the acceptable materials of the heat exchanger.

A temperature difference of 50°C or more between the tubes and the shell will probably pose *differential thermal expansion* problems and needs to be considered. In the case of corrosive fluids, we may have to select expensive *corrosion-resistant* materials such as stainless steel or even titanium if we are not willing to replace low-cost heat exchangers frequently.

Other Considerations

There are other considerations in the selection of heat exchangers that may or may not be important, depending on the application. For example, being *leak-tight* is an important consideration when *toxic* or *expensive* fluids are involved. Ease of servicing, low maintenance cost, and safety and reliability are some other important considerations in the selection process. Quietness is one of the primary considerations in the selection of liquid-to-air heat exchangers used in heating and air-conditioning applications.

EXAMPLE 10-10 Installing a Heat Exchanger to Save Energy and Money

In a dairy plant, milk is pasteurized by hot water supplied by a natural gas furnace. The hot water is then discharged to an open floor drain at 80°C at a rate of 15 kg/min. The plant operates 24 h a day and 365 days a year. The furnace has an efficiency of 80 percent, and the cost of the natural gas is $0.40 per therm (1 therm = 105,500 kJ). The average temperature of the cold water entering the

furnace throughout the year is 15°C. The drained hot water cannot be returned to the furnace and recirculated, because it is contaminated during the process.

In order to save energy, installation of a water-to-water heat exchanger to preheat the incoming cold water by the drained hot water is proposed. Assuming that the heat exchanger will recover 75 percent of the available heat in the hot water, determine the heat transfer rating of the heat exchanger that needs to be purchased and suggest a suitable type. Also, determine the amount of money this heat exchanger will save the company per year from natural gas savings.

Solution A water-to-water heat exchanger is to be installed to transfer energy from drained hot water to the incoming cold water to preheat it. The rate of heat transfer in the heat exchanger and the amount of energy and money saved per year are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The effectiveness of the heat exchanger remains constant.

Properties We use the specific heat of water at room temperature, $C_p = 4.18$ kJ/kg · °C (Table A-9), and treat it as a constant.

Analysis A schematic of the prospective heat exchanger is given in Figure 10-31. The heat recovery from the hot water will be a maximum when it leaves the heat exchanger at the inlet temperature of the cold water. Therefore,

$$\begin{aligned}\dot{Q}_{max} &= \dot{m}_h C_p (T_{h,in} - T_{c,in}) \\ &= \left(\frac{15}{60}\text{ kg/s}\right)(4.18\text{ kJ/kg} \cdot \text{°C})(80 - 15)\text{°C} \\ &= 67.9\text{ kJ/s}\end{aligned}$$

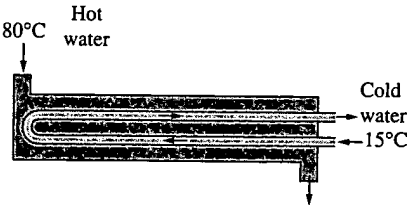


FIGURE 10-31
Schematic for Example 10-10.

That is, the existing hot water stream has the potential to supply heat at a rate of 67.9 kJ/s to the incoming cold water. This value would be approached in a counter-flow heat exchanger with a *very large* heat transfer surface area. A heat exchanger of reasonable size and cost can capture 75 percent of this heat transfer potential. Thus, the heat transfer rating of the prospective heat exchanger must be

$$\dot{Q} = \varepsilon \dot{Q}_{max} = (0.75)(67.9\text{ kJ/s}) = \mathbf{50.9\ kJ/s}$$

That is, the heat exchanger should be able to deliver heat at a rate of 50.9 kJ/s from the hot to the cold water. An ordinary plate or *shell-and-tube* heat exchanger should be adequate for this purpose, since both sides of the heat exchanger involve the same fluid at comparable flow rates and thus comparable heat transfer coefficients. (Note that if we were heating air with hot water, we would have to specify a heat exchanger that has a large surface area on the air side.)

The heat exchanger will operate 24 h a day and 365 days a year. Therefore, the annual operating hours are

$$\text{Operating hours} = (24\text{ h/day})(365\text{ days/year}) = 8760\text{ h/year}$$

Noting that this heat exchanger saves 50.9 kJ of energy per second, the energy saved during an entire year will be

$$\begin{aligned}\text{Energy saved} &= (\text{Heat transfer rate})(\text{Operation time}) \\ &= (50.9\text{ kJ/s})(8760\text{ h/year})(3600\text{ s/h}) \\ &= 1.605 \times 10^9\text{ kJ/year}\end{aligned}$$

The furnace is said to be 80-percent efficient. That is, for each 80 units of heat supplied by the furnace, natural gas with an energy content of 100 units must be supplied to the furnace. Therefore, the energy savings determined above result in fuel savings in the amount of

$$\text{Fuel saved} = \frac{\text{Energy saved}}{\text{Furnace efficiency}} = \frac{1.605 \times 10^9 \text{ kJ/year}}{0.80}\left(\frac{1 \text{ therm}}{105{,}500 \text{ kJ}}\right)$$
$$= 19{,}020 \text{ therms/year}$$

since 1 therm = 105,500 kJ. Noting that the price of natural gas is $0.40 per therm, the amount of money saved becomes

$$\text{Money saved} = (\text{Fuel saved}) \times (\text{Price of fuel})$$
$$= (19{,}020 \text{ therms/year})(\$0.40/\text{therm})$$
$$= \mathbf{\$7607/year}$$

Therefore, the installation of the proposed heat exchanger will save the company $7607 a year, and the installation cost of the heat exchanger will probably be paid from the fuel savings in a short time.

10-7 ■ SUMMARY

Heat exchangers are devices that allow the exchange of heat between two fluids without allowing them to mix with each other. Heat exchangers are manufactured in a variety of types, the simplest being the *double-pipe* heat exchanger. In a *parallel-flow* type, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction, whereas in a *counter-flow* type, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite directions. In *compact* heat exchangers, the two fluids move perpendicular to each other, and such a flow configuration is called *cross-flow*. Other common types of heat exchangers in industrial applications are the *plate* and the *shell-and-tube* heat exchangers.

Heat transfer in a heat exchanger usually involves convection in each fluid and conduction through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an *overall heat transfer coefficient U* or a *total thermal resistance R*, expressed as

$$\frac{1}{UA} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + R_{\text{wall}} + \frac{1}{h_oA_o}$$

where the subscripts i and o stand for the inner and outer surfaces of the wall that separates the two fluids, respectively. When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, the above relation simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where $U \approx U_i \approx U_o$. The effects of fouling on both the inner and the outer surfaces of the tubes of a heat exchanger can be accounted for by

$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of the inner and outer surfaces and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.

In a well-insulated heat exchanger, the rate of heat transfer from the hot fluid is equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = C_c(T_{c,\text{out}} - T_{c,\text{in}})$$

and

$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,\text{in}} - T_{h,\text{out}}) = C_h(T_{h,\text{in}} - T_{h,\text{out}})$$

where the subscripts c and h stand for the cold and hot fluids, respectively, and the product of the mass flow rate and the specific heat of a fluid $C = \dot{m}C_p$ is called the *heat capacity rate*.

Of the two methods used in the analysis of heat exchangers, the *log mean temperature difference* (or LMTD) method is best suited for determining the size of a heat exchanger when all the inlet and the outlet temperatures are known. The *effectiveness-NTU* method is best suited to predict the outlet temperatures of the hot and cold fluid streams in a specified heat exchanger. In the LMTD method, the rate of heat transfer is determined from

$$\dot{Q} = UA\Delta T_{\text{lm}}$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

is the *log mean temperature difference*, which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature differences between the two fluids at the two ends (inlet and outlet) of the heat exchanger. For cross-flow and multipass shell-and-tube heat exchangers, the logarithmic mean temperature difference is related to the counter-flow one $\Delta T_{\text{lm},CF}$ as

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm},CF}$$

where F is the *correction factor*, which depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams.

The *effectiveness* of a heat exchanger is defined as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

where

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})$$

and $C_{\min}$ is the smaller of $C_h = \dot{m}_h C_{ph}$ and $C_c = \dot{m}_c C_{pc}$. The effectiveness of heat exchangers can be determined from effectiveness relations or charts.

The selection
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type, materials, ar

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The selection or design of a heat exchanger depends on several factors such as the heat transfer rate, cost, pressure drop, size, weight, construction type, materials, and operating environment.

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